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MOREY'S ARITHMETICS

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ADVANCED ARITHMETIC

PART TWO

CHARLES SCRIBNER'S SONS

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MOREY'S ARITHMETICS

ADVANCED ARITHMETIC

BY

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PART TWO

NEW YORK
CHARLES SCRIBNER'S SONS
1911

Educ T 119.591 Pt. 2

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PREFACE

THE object of the series of text-books of which Advanced Arithmetic is a part is threefold: —

I. To secure accuracy and facility in the mechanics of number. To accomplish this result there is a continuous presentation of the fundamental processes with integers and with fractions, both common and decimal.

II. To relate the use of number to the affairs of everyday life. This requires the solution of problems in household and school economics, in the affairs of the playground, the workshop, and the farm, and a study of conditions existing in the business world.

III. To develop a power of insight and analysis which shall render pupils resourceful and self-reliant. This involves the introduction of many problems requiring the use of more than a single process in their solution, and drawn from a great variety of sources, both within and without the experience of the pupil. A foundation is thus laid for work in mathematics beyond grammar school grades.

Some subjects ordinarily included in grammar school courses are purposely omitted because it is believed that the mastery of a few subjects is the sane and wise method of procedure in the development of mathematical power. Quality, not quantity, of work is of prime importance in training young pupils.

The subject matter of Advanced Arithmetic is so arranged that pupils leaving school before the completion of the regular course are given opportunity to acquire some knowledge of practical business methods.

The author wishes to acknowledge his indebtedness to all who have assisted in the preparation of the manuscript, and especially to Mr. Myron T. Pritchard, Master of the Everett School, Boston, Massachusetts, for wise counsel and criticism.

C. W. M.

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PART II

ADVANCED ARITHMETIC

NOTATION AND NUMERATION

1. In our system of writing numbers, how many units of one order are required to make one unit of the next higher order?

2. What name is given to such a system of notation?

3. How many places are required to write ten in figures? One thousand? One million?

Using the symbols 4, 6, 8, 3, 2, write:

4. The largest number possible. Read it.

5. The smallest number possible. Read it.

Separate into groups, read aloud, and then write in words:

6. 507 **7.** 8700 **8.** 90376 **9.** 299000
10. 54000 **11.** 50040 **12.** 7068 **13.** 18043
14. 6073 **15.** 600708 **16.** 70007 **17.** 6007
18. 4061295 **19.** 9007005 **20.** 123456789 **21.** 870642083

Read:

22. The quantity of peanuts exported from the United States in one year was: to Oceania, 789 pounds; to Asia, 1100 pounds; to Africa, 66,443 pounds; to South America, 334,992; to Europe, 1,497,625 pounds; to North America, 3,600,158 pounds.

23. In our country in a certain year there were 8244 establishments engaged in book and job printing with a total capital of $142,015,638. These establishments employed 87,746 persons and paid in wages $48,720,854. The total cost of materials used was $52,575,110, and the total value of finished products was $182,611,720.

Write in figures :

1. Five thousand seventy-two.

2. Forty thousand nine hundred eighty.

3. Eight hundred sixty-seven thousand five hundred eight.

4. Two hundred thousand.

5. Two hundred two thousand.

6. Two hundred thousand two hundred.

7. Two hundred thousand two.

8. Eighty-six million twenty-seven thousand twenty four.

9. Nine hundred seventy-eight million three hundred fifty-seven thousand seven hundred sixty-four.

10. Sixty million sixty thousand sixty.

ROMAN NOTATION AND NUMERATION

The Roman system of notation uses the following letters:

I V X L C D M

1. What value is assigned to each ?

2. Read: II III XX XXX CC CCC MM

3. What effect is produced by repeating a letter?

4. Read: VI LXI LX CX DC MC

5. What effect is produced by *annexing* a letter to one of greater value?

6. Read: IV IX XL XC CD CM

7. What effect is produced by *prefixing* a letter to one of greater value?

A dash over a letter increases its value a thousandfold.

Thus, $\overline{XX} = 20{,}000$; $\overline{C} = 100{,}000$.

8. Read: XL $\overline{XL}$ LXX $\overline{LXX}$ D $\overline{D}$

9. Write these letters so that they shall have one thousand times their ordinary value: I V X L C M

10. Write 1492 in Roman notation.

1492 = 1000 + 400 + 90 + 2
M + CCCC + XC + II = MCCCCXCII

Write in letters:

11. 36 **12.** 47 **13.** 63 **14.** 72 **15.** 89
16. 99 **17.** 900 **18.** 909 **19.** 990 **20.** 999
21. 1789 **22.** 1850 **23.** 1910 **24.** 1915 **25.** 1920

26. Write MDCCLXXXIII in Arabic notation.

MDCCLXXXIII = M + DCC + LXXX + III
1000 + 700 + 80 + 3 = 1783

Write in figures:

27. XIV **28.** XXXIX **29.** LXXIX
30. MCMX **31.** MDCCC **32.** MDCCLXXIII
33. MDCCCXC **34.** MCMXII **35.** MCMXIX

36. On the front page of a magazine is printed "Volume XLVIII, Number 5." What does it mean?

37. Elizabeth has a book bearing on its title page the letters MDCCCXII. In what year was it published?

38. Clarence has a foreign coin stamped MDCCLXX. In what year was it minted?

39. On the corner stone of a public library are these letters — MDCCCXCII. When was the corner stone laid?

40. The seal of a celebrated boys' school bears the letters MDCCLXVIII. When was the school founded?

41. A monument commemorating the landing of the Pilgrims bears these dates: MDCXX MCMX The upper date tells when the Pilgrims landed; the lower, when the monument was dedicated. Write both dates in figures.

42. Write in letters the two dates which should be inscribed on a tablet commemorating the birth of Washington (1732) and the 200th anniversary of his birth.

43. Write (1) in words; (2) in figures; (3) in letters:

Your age; your grade in school; the number of your schoolroom; the number of pupils in the room; this page of the book; the number of pages in the book; the present month of the year; the day of the month; the present year; the street number of your house; the year in which you were born.

DICTATION EXERCISES

1. 64 ÷ 8, × 3, + 11, ÷ 7, – 2, × 9, + 6, ÷ 3, × 5.
2. 12 × 9, + 2, ÷ 11, × 4, – 8, × 2, – 8, ÷ 7, × 5, + 15.
3. 7 + 6, – 3, × 7, + 2, ÷ 8, – 3, × 6, + 4, ÷ 10, – 4.
4. 12 × 7, – 9, ÷ 5, + 6, ÷ 7, × 13, + 3, ÷ 6, + 4, × 3.
5. 108 ÷ 9, + 8, – 4, × 6, ÷ 8, + 8, – 5, × 3, ÷ 9, + 5.
6. 64 ÷ 4, – 2, × 2, ÷ 4, + 4, × 8, – 4, ÷ 7, – 6, × 0.

Note. Similar dictations should be given daily.

ADDING TWO COLUMNS AT ONCE

Find the sum, adding both columns at once; test the work:

1. 37	**2.** 56	**3.** 28	**4.** 43	**5.** 54	**6.** 72
7	8	9	9	7	8
14	4	5	8	5	3
6	13	3	7	6	4
3	5	2	25	5	2
28	8	17	3	9	17
5	29	5	2	22	6
9	7	26	16	7	35
2	4	8	4	6	8
2	3	4	1	15	2
3	2	1	4	5	3

7. 11	**8.** 15	**9.** 7	**10.** 8	**11.** 10	**12.** 6
5	4	3	5	7	12
6	7	12	40	6	7
10	2	8	2	25	20
3	20	34	4	2	10
22	8	9	29	3	5
7	3	6	7	37	42
12	11	20	18	1	8
9	7	7	3	8	3
5	2	14	11	5	27
4	5	6	8	7	2

Beginning with 25, or any given number, add the numbers in examples 1 to 12.

Note. Drill in this form of addition should be given frequently until pupils become proficient.

DRILL TABLE: FUNDAMENTAL PROCESSES

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
1.	144	128	190	178	135	108	125	162
2.	138	191	127	107	179	136	106	156
3.	192	126	137	173	145	180	157	124
4.	129	193	174	130	104	158	181	146
5.	163	109	194	142	159	147	105	182
6.	131	175	155	199	103	134	183	123
7.	176	110	164	132	198	184	154	161
8.	143	133	148	165	185	197	122	153
9.	111	170	113	186	166	121	196	115
10.	149	112	187	140	152	167	116	195
11.	160	188	119	172	139	150	168	141
12.	189	118	151	117	120	171	114	169

EXERCISES IN ADDITION

To each number in the table add 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12.

To each number add 10; 20; 30; 40; 50; 60; 70; 80; 90; 110; 120.

Give the sum of each number in the table and 79. Thus, 144 + 79; 144 + 70 + 9; think 214, 223; say 223.

Add other numbers to the numbers in the table.

Find the sum of column *A*. Of the other columns.

Find the sum of row 1. Of the other rows.

EXERCISES IN SUBTRACTION

From the numbers in the drill table take 2; 4; 6; 8; 1; 3; 5; 7; 9.

Subtract each number in the drill table from 200; from 500; from 800.

Take 69 from each number in the drill table. Thus, 144 − 69 = 144 − 60 − 9; think 84, 75; say 75.

Find the difference between other numbers and the numbers in the drill table.

1. From 865 take 376.
2. Take 589 from 706.
3. From 407 take 69.
4. Take 418 from 724.
5. From 5000 take 976.
6. Take 1987 from 5086.
7. Fom 4652 take 848.
8. Take 1695 from 4763.
9. From 8507 take 3698.
10. Take 6007 from 70,006.
11. From 65,089 take 9999.
12. Take 40,004 from 90,000.
13. From 60,057 take 39,068.
14. Take 9728 from 69,573.
15. From 46,008 take 17,439.
16. Take 80,668 from 861,547.
17. From 700,000 take 365,746.
18. Take 23,994 from 470,083.
19. From 248,576 take 150,798.
20. Take 476,059 from 826,045.

EXERCISES IN MULTIPLICATION

Multiply (orally) each number in the drill table on page 146 by 2; 3; 4; 5; 6; 7; 8; 9.

Multiply:

1. 65 by 47
2. 83 by 96
3. 70 by 68
4. 94 by 57
5. 76 by 86
6. 573 by 64
7. 609 by 97
8. 863 by 168
9. 705 by 308
10. 965 by 86
11. 867 by 397
12. 905 by 96
13. 4809 by 73
14. 6000 by 705
15. 3456 by 98
16. 7008 by 807
17. 4365 by 1729
18. 9046 by 378
19. 49,308 by 67
20. 38,067 by 209

21. 86,297 by 56
22. 73,009 by 2376
23. 96,580 by 374
24. 63,084 by 1708
25. 46,376 by 7236
26. 56,473 by 476
27. 743,618 by 97
28. 360,874 by 365
29. 426,083 by 25,007
30. 367,589 by 24,387

EXERCISES IN DIVISION

Divide (orally) the numbers in the drill table on page 146 by 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12.

I. Divide (short division) each dividend in the examples below by 2; 3; 4; 5; 6; 7; 8; 9; 10; 11; 12.

II. Divide:

1. 638 by 47
2. 809 by 38
3. 793 by 67
4. 985 by 73
5. 807 by 96
6. 986 by 463
7. 879 by 164
8. 6844 by 97
9. 4009 by 73
10. 3967 by 289
11. 4368 by 48
12. 6840 by 375
13. 9000 by 173
14. 4738 by 294
15. 36,900 by 75
16. 67,857 by 89
17. 37,008 by 96
18. 65,078 by 359
19. 39,872 by 623
20. 46,798 by 604
21. 84,376 by 356
22. 730,657 by 73
23. 496,320 by 475
24. 327,009 by 764
25. 510,936 by 732
26. 460,738 by 326
27. 645,170 by 866
28. 306,752 by 1345
29. 3,194,658 by 456
30. 4,073,251 by 2375

WRITTEN PROBLEMS

1. The number of immigrants to the United States for the year ending June 30, 1909, was as follows: Landing at Baltimore, 18,966; at Boston, 36,318; at New York, 580,617; at Philadelphia, 14,294; at San Francisco, 3103; at all other ports, 98,488. Find the total number.

2. Of the 654,875 immigrants to the United States from Europe in the same year, 170,191 came from Austria; 25,540 from Germany; 14,111 from Greece; 183,218 from Italy; 13,627 from Norway; 120,460 from Russia; 14,474 from Sweden; and 71,826 from the United Kingdom. What was the total number from these countries? What was the total number from European countries not mentioned?

3. A hardware merchant estimated the value of stock on hand January 1, at $14,813.76. During the year he bought goods to the value of $5817.56, and sold goods to the value of $12,627.98. What was the value of the stock on hand December 31?

EXPORTS OF BOOTS AND SHOES FROM THE UNITED STATES IN FIVE YEARS

EXPORTED TO	1905	1906	1907	1908	1909
	Pairs	Pairs	Pairs	Pairs	Pairs
Europe . . .	1,149,899	1,257,004	1,215,428	1,241,649	1,235,670
North America	3,425,111	3,774,655	3,833,856	4,445,141	4,410,550
South America	206,117	197,549	229,478	266,347	229,387
Asia	42,332	47,074	35,766	40,242	30,178
Oceania . . .	362,646	312,869	460,469	490,566	205,871
Africa . . .	129,594	83,098	58,917	68,467	64,923

Exported to	1905	1906	1907	1908	1909
	Dollars	Dollars	Dollars	Dollars	Dollars
Europe . . .	2,472,608	2,591,144	3,062,988	3,154,107	3,021,711
North America	4,238,642	5,367,349	6,167,179	6,660,657	6,183,859
South America	325,467	363,310	458,613	548,702	474,119
Asia	77,277	80,868	54,782	85,988	67,849
Oceania . . .	673,056	552,104	792,464	850,449	408,096
Africa . . .	270,647	187,973	130,923	169,656	150,180

4. Find the total number of pairs exported in 1905; their total value.

5. Find the same facts in regard to each of the other years.

6. What was the total number of pairs exported to Europe in the five years? Their total value?

7. Find the total shipments, and their total value, to each of the other parts of the world in the five years.

8. A merchant bought 12 dozen straw hats at 28 cents apiece and sold them at 45 cents each. How much did he gain?

9. Oscar sold 18 chickens and 14 doves for $7.30. He sold the doves at 20 cents apiece. How much did he get apiece for the chickens?

10. Mr. Spare bought from various farmers 24 cases of eggs, 30 dozen to a case, at 24 cents a dozen. He sold the lot for $187.20. What was his profit on a dozen?

11. The vote for governor in a state election was as follows:

A. B., Democrat, received 229,352 votes; C. D., Republican, 194,173 votes; E. F., Socialist, 11,396 votes;

G. H., Prohibition, 3277 votes; I. J., Socialist Labor, 2613 votes; all others, 20 votes.

Find the total vote. By what plurality over C. D. was A. B. elected?

12. A wholesale merchant bought 360 barrels of apples at $1.12 per barrel. After sorting them, he had 325 barrels which he sold at $2.40 per barrel. How much did he gain?

13. The Mississippi River is 2775 miles in length and is navigable for 2429.5 miles from its mouth. For how many miles is it not navigable?

14. In going from Vicksburg, Miss., to Memphis, Tenn., a boat travels 369 miles. If Vicksburg is 362 miles above New Orleans, and Memphis is 432 miles below St. Louis, how many miles is it by water from St. Louis to New Orleans?

15. At the rate of 37½ cents each, how much must be paid for one and a half dozen lace collars?

16. At the rate of 8 for a dollar, how many handkerchiefs can be bought for 75 cents? For $1¼?

17. An encyclopedia of 8 volumes bought for $56 was sold at the rate of $2.85 per volume. What was the loss?

18. A catch of 70,000 pounds of codfish was sold at $3.65 per hundredweight. After deducting the expense of the trip, $345, the balance was divided as follows: ⅙ to the owners of the vessel, and the remainder equally among the crew of 21 men. What was each man's share?

19. A merchant buys neckties at the rate of 6 for a dollar and sells them for 25 cents apiece. What is the gain on a dozen?

20. A man earning $2.56 per day of 8 hours lost 12½ hours' time in a week. What were his wages for that week?

21. An estate worth $23,274 was divided as follows: ⅓ to the mother, and the remainder equally among 4 children. How much did the mother receive? How much did each child receive?

22. Frank raised 120 cucumbers which he sold at the rate of 2 for 5 cents. How much did he receive?

23. What is the difference in the amounts received in a year by two men, one of whom receives $75 a month, and the other 35 cents an hour, working 8 hours a day, 308 days in the year?

24. If the average yearly expense to a city for each pupil in the kindergarten is $27.18, in the primary school $16.58, in the grammar school $19.09, and in the high school $40.68, what is the total cost of educating a child who attends the kindergarten 2 years, the primary school 3 years, the grammar school 5 years, and the high school 4 years?

25. A test of a herd of cows showed that the 5 poorest ate feed worth $140 and returned $143 in product, while the 5 best ate feed worth $204 and returned $395. What was the average profit per cow of the 5 poorest? Of the 5 best? Of the whole herd?

26. A Maine farmer raised 2100 bushels of potatoes on 7 acres of land at a cost of $80 an acre, and sold his crop at $1½ per barrel of 2½ bushels. Find the profit per acre.

27. Another farmer in Maine harvested 6000 barrels of potatoes from 50 acres at a cost of $60 an acre, and sold them at $1½ per barrel. Find his profit per acre.

28. If the average potato yield in the United States is 100 bushels per acre, find how many times the average yield was produced per acre in problem 26; in problem 27.

29. Fruit merchants in St. Louis bought the entire apple crop — 15,760 barrels — of a town in Massachusetts at $1.75 per barrel. How much was paid?

30. Mr. Franklin received $300 from the sale of the crop from 25 apple trees. If the selling price was $1.75 per barrel, what was the average yield per tree?

31. Mr. Peterson bought 28 acres of land for $630, and another lot at the same rate per acre for $945. How many acres in the second lot?

32. At an agricultural experiment station a flock of 63 Merino sheep was fed in 110 days 2,785 pounds of oats, 3,135 pounds of corn, 514 pounds of bran, 324 pounds of oil meal, 743 pounds of mangels, and 14,360 pounds of hay. Find the total number of pounds of feed consumed; the average number of pounds of feed consumed by each sheep; the average number of pounds of feed consumed each day.

UNITED STATES MONEY *Oral*

1. How many half dollars in $6? How many quarters? Dimes? Nickels? Cents?

2. How many half dollars in $7.50? Quarters? Dimes? Nickels? Cents?

3. Express as dollars: 19 half dollars 25 quarters 57 dimes 63 nickels 206 cents 56 cents 7 cents

4. Inez paid a bill with dimes, nickels, and cents, using 10 dimes, one half as many nickels, and one fifth as many cents. What was the amount of the bill?

Solve:

	A	*B*	*C*
5.	$4 − $0.10	4 × $0.10	$4 ÷ $0.10
6.	$6 − $0.20	6 × $0.20	$6 ÷ $0.20
7.	$16 − $0.25	16 × $0.25	$16 ÷ $0.25
8.	$48 − $0.50	48 × $0.50	$48 ÷ $0.50
9.	$30 − $$0.33\frac{1}{3}$	30 × $$0.33\frac{1}{3}$	$30 ÷ $$0.33\frac{1}{3}$
10.	$24 − $$0.16\frac{2}{3}$	24 × $$0.16\frac{2}{3}$	$24 ÷ $$0.16\frac{2}{3}$
11.	$2 − $$0.08\frac{1}{3}$	2 × $$0.08\frac{1}{3}$	$2 ÷ $$0.08\frac{1}{3}$
12.	$3 − $$0.12\frac{1}{2}$	3 × $$0.12\frac{1}{2}$	$3 ÷ $$0.12\frac{1}{2}$
13.	$4 − $$0.06\frac{1}{4}$	4 × $$0.06\frac{1}{4}$	$4 ÷ $$0.06\frac{1}{4}$

Make up problems based on the above exercise.

PURCHASES AT THE MARKET *Oral*

Find the cost of:

1. $2\frac{1}{2}$ lb. lamb @ 15¢.

$2\frac{1}{2}$ times 15¢ = $37\frac{1}{2}$¢. When the amount of a purchase contains a fraction of a cent equal to $\frac{1}{2}$, the next full cent is charged. The purchaser in this case would pay 38 cents.

2. $1\frac{1}{2}$ lb. sausage @ 17¢
3. $6\frac{1}{2}$ lb. beef @ 23¢
4. $2\frac{1}{4}$ lb. ham @ 23¢
5. $3\frac{1}{2}$ lb. veal @ 19¢
6. $1\frac{3}{4}$ lb. steak @ 30¢
7. $1\frac{1}{2}$ lb. haddock @ 11¢
8. $2\frac{1}{2}$ lb. bluefish @ 17¢
9. $1\frac{1}{2}$ lb. bacon @ 25¢
10. $3\frac{3}{8}$ lb. chicken @ 28¢
11. $6\frac{1}{2}$ lb. tomatoes @ 5¢
12. $2\frac{1}{2}$ doz. oranges @ 25¢
13. $1\frac{1}{2}$ pk. apples @ 37¢
14. $2\frac{1}{2}$ doz. eggs @ 35¢
15. $1\frac{3}{4}$ doz. lemons @ 25¢
16. $2\frac{1}{4}$ lb. tongue @ 23¢
17. $2\frac{3}{4}$ lb. lobster @ 25¢
18. $1\frac{1}{4}$ lb. dried beef @ 15¢
19. $4\frac{1}{2}$ lb. mutton @ 17¢
20. $1\frac{1}{2}$ lb. carrots @ 3¢
21. $2\frac{1}{4}$ lb. parsnips @ 3¢

SHORT METHODS OF MULTIPLYING

1. Find the cost of 328 yards of muslin at 25 cents a yard.

SOLUTION. At $1 a yard the cost would be $328. At 25 cents a yard the cost will be ¼ of $328, or $82.

In the following problems use pencil only in writing the cost of each quantity and the total cost.

Find the total cost of:

2. 325 lb. @ 10¢
600 lb. @ 25¢
240 lb. @ 12½¢
640 lb. @ 6¼¢

3. 148 lb. @ 50¢
360 lb. @ 33⅓¢
246 lb. @ 16⅔¢
480 lb. @ 8⅓¢

4. 25 yd. @ 20¢
60 yd. @ 40¢
45 yd. @ 60¢
75 yd. @ 80¢

5. 145 yd. @ 10¢
240 yd. @ 30¢
150 yd. @ 70¢
120 yd. @ 90¢

6. 216 doz. @ 33⅓¢
360 doz. @ 66⅔¢
108 doz. @ 16⅔¢
120 doz. @ 83⅓¢

7. 320 doz. @ 12½¢
560 doz. @ 37½¢
640 doz. @ 62½¢
720 doz. @ 87½¢

8. 60 yd. @ 3⅛¢
80 yd. @ 2½¢
64 yd. @ 6¼¢
72 yd. @ 8⅓¢

9. 56 yd. @ 12½¢
96 yd. @ 33⅓¢
84 yd. @ 8⅓¢
45 yd. @ 40¢

10. 15 yd. @ 50¢
25 yd. @ 25¢
10 yd. @ 75¢
40 yd. @ 62½¢
96 yd. @ 8⅓¢

11. 80 yd. @ 60¢
72 yd. @ 83⅓¢
72 yd. @ 37½¢
36 yd. @ 33⅓¢
29 yd. @ 25¢

POSTAL PROBLEMS

DOMESTIC RATES OF POSTAGE

The following are the rates to all parts of the United States, including Panama Canal Zone, Porto Rico, Guam, Hawaiian Islands, Philippines, Tutuila (Samoan Islands), Canada, Cuba, Mexico, Republic of Panama, and Shanghai, China.

First-class Matter: Letters and all sealed matter, 2 cents an ounce or fraction thereof.

Second-class Matter: Newspapers and periodicals sent by publishers, 1 cent a pound or fraction thereof; when sent by others, 1 cent for four ounces or fraction thereof.

Third-class Matter: Books, circulars, pictures, etc., 1 cent for two ounces or fraction thereof.

Fourth-class Matter: Merchandise and all matter not included in any of the other classes, 1 cent an ounce, except seeds, which may be sent at the rate of 1 cent for two ounces.

Registration Fee (to insure safe delivery): 10 cents in addition to regular postage.

Special Delivery Fee (to insure quick delivery): 10 cents in addition to regular postage.

1. What is the postage on a letter weighing $\frac{1}{2}$ oz.? $\frac{3}{4}$ oz.? $1\frac{3}{4}$ oz.? $2\frac{1}{4}$ oz.? 3 oz.? $3\frac{1}{2}$ oz.? 4 oz.? $4\frac{1}{4}$ oz.?

2. Mr. Chase sends by registered mail a letter weighing $2\frac{1}{2}$ ounces. What is the postage?

3. Mr. Luther sends a half-ounce letter by special delivery to his son. What is the postage?

4. A merchant sends in one day 37 letters (5 by special delivery) weighing $\frac{1}{2}$ ounce each, 350 catalogues each weighing $2\frac{3}{4}$ ounces, and 36 packages of samples each weighing $1\frac{1}{2}$ ounces. Find the amount of postage required.

5. A manufacturer sends out 1500 circulars weighing $1\frac{1}{2}$ ounces each, and under separate covers the same number of samples weighing $2\frac{1}{2}$ ounces each. What postage did he pay?

6. A magazine publisher sends by mail one and three quarters tons of magazines each month. What is his yearly bill for postage?

7. Myrtle sent 12 Christmas cards (sealed), 15 postal cards, 2 packages each weighing 5 ounces, 1 package weighing ¾ of a pound, and a book weighing 15 ounces. How much did she pay for postage?

8. What postage is paid by a newspaper company on 8 copies of its daily paper if each paper weighs 4 ounces? What postage must Thomas pay if he mails 8 copies of the same paper to his friends?

9. If a copy of a monthly magazine weighs 12½ ounces, what postage must the publisher pay on 800,000 copies?

CASH ACCOUNTS

In cash accounts, "Cash" may be regarded as a person who accounts for one's money. Cash is debtor to money received, and credited with money paid out.

Dr. CASH Cr.

1911 Mar. 1	On hand	$ 3	00	1911 Mar. 3	By Shoes	$ 3	50
" 8	To Allowance	15	00	" 10	By Books	12	50
" 9	To Posters	4	50	" 14	By Sweater	4	75
" 25	To Present	5	00	" 16	By Subscription	2	00
				" 21	By Class Dues	3	00
				" 31	By Balance	1	75
		27	50			27	50
1911 Apr. 1	On hand	1	75				

Write these accounts in proper form:

1. Cash account of Clara Dow: Receipts: Mar. 1, 1911, on hand $0.65; Mar. 4, monthly allowance $4.00;

Mar. 11, old gold $3.50; Mar. 21, birthday present $5.00. Expenses: Mar. 2, flowers $0.50; Mar. 7, book $1.50; Mar. 10, fruit $0.85; Mar. 17, charity $0.60; Mar. 21, concert $0.75; Mar. 23, bag $3.00; Mar. 28, pin $4.25.

2. Cash account of Rufus Brown: Receipts: Mar. 1, 1911, on hand $65.00; Mar. 8, month's pay at $30.00 a week; Mar. 10, rent from house $20.00. Expenses: Mar. 4, taxes $75.00; Mar. 9, house expenses $45.00; Mar. 17, clothing $50.00; Mar. 23, repairs $5.00.

3. Cash account of a music teacher: Receipts: Mar. 1, 1911, on hand $8.50; Mar. 2, from 5 pupils, 10 lessons each, at $2.00 a lesson; Mar. 10, from 3 pupils, 20 lessons each, at $2.00 a lesson; Mar. 18, from 4 pupils, 10 lessons each, at $1.50 a lesson. Expenses: Mar. 4, living expenses $25.00; Mar. 10, tuning pianos $5.00; Mar. 15, special instruction, 10 lessons at $5.00 each.

4. Cash account of Mrs. Green: Receipts: Mar. 1, 1911, on hand $10.00; Mar. 4, rent from rooms $32.00; Mar. 15, pension $12.00; Mar. 18, sewing $9.00; Mar. 25, board $16.00. Expenses: Mar. 6, groceries $15.00; Mar. 7, provisions $18.00; Mar. 11, milk $2.80; Mar. 15, gas $4.50; Mar. 21, clothing $12.50.

5. Cash account of a dressmaker: Receipts: Mar. 1, 1911, on hand $10.50; Mar. 7, making 2 gowns at $15.00 each; Mar. 9, trimmings $5.80; Mar. 16, making waist $6.00; Mar. 21, cloth and making gown $28.00; Mar. 28, making 2 gowns at $18.50 each. Expenses: Mar. 4, seamstress 24 days at $1.50 a day; Mar. 17, silk and trimmings $10.00; Mar. 24, buttons and braid $4.50; Mar. 30, rent $30.00; Mar. 31, linings $3.25.

EXPENDITURES OF A PARK COMMISSION

Table showing the Amounts expended by a Park Commission for Maintenance and Improvement in One Year

	Name	Maintenance		Improvement	
		Labor	Material	Labor	Material
1.	North Common	$ 672.54	$ 50.57	$ 4.31	$ 31.80
2.	South Common	911.02	46.37		106.92
3.	Fort Hill Park	2272.30	402.11	3575.77	1667.51
4.	Belvidere Park	115.66	35.63		
5.	Tyler Park	133.79	22.84		
6.	Mt. Vernon Park	45.21	10.68		
7.	Monument Square	34.23	37.27	16.70	21.12
8.	City Hall and Memorial Building	125.34	28.78		
9.	Lincoln Playground	16.27		112.97	27.00
10.	Hovey Square	4.63	4.87		
11.	Mansur Square	5.94	12.92		
12.	Colburn Square			2.35	
13.	Lenox Square	16.53		57.11	54.56
14.	Princeton Square	10.28		7.70	18.63
15.	Middlesex Square	6.02		17.24	32.46
16.	Fayette Street Playground	6.56			
17.	Varnum Avenue River Bank	13.35	5.00		
18.	Colonial Avenue River Bank	29.02	8.69		
19.	Glacial Rock				39.60
20.	Water Works Square	6.52	.52	29.21	38.99
21.	Penniman Square	8.02	12.94		
22.	Varnum Square	10.59	22.33	10.69	5.90
23.	Lincoln Square	1.15		89.79	266.00
24.	Stable	106.94	1611.90	19.30	2.20
25.	Carpenter shop	163.01	73.74		
26.	Office Expenses	1344.66	345.93		

1. Find the total amount expended for labor in maintenance.

2. Find the total amount expended for material in maintenance.

3. Find the total amount expended for maintenance.

4. What was the total amount expended for labor in improvement?

5. What was the total amount expended for material in improvement?

6. What was the total amount expended for improvement?

7. What is the sum of answers to **3** and **6**? What does it represent?

8. What was the total amount expended on North Common? On each of the other parks? For other purposes?

9. Find the sum of the amounts expended.

10. Compare the answer to **9** and the answer to **7**.

REVIEW EXERCISE

1. Which of these numbers are concrete? Abstract?

7 men   3 $5   17   19 days   150

2. What is an abstract number? A concrete number?

3. Which of these numbers are prime? Composite?

42   37   56   63   19   39   61   84   47

4. What is a prime number? A composite number?

5. What are the factors of 24? 56? 84? 120?

6. What is meant by the factors of a number?

7. What are the prime factors of 56? 96? 144?

8. What is a prime factor?

9. What number is represented by $2^3 \times 5$? By $4^2 \times 3^2$? By $5^3 \times 2^2$?

10. What does the exponent of a number tell?

11. What is the greatest common divisor of 12, 15, and 18? Of 24, 36, 60? Of 28, 35, 56?

12. What is the length of the longest stick that will exactly measure distances of either 15 feet, 18 feet, or 30 feet?

13. What is the least common multiple of 9, 12, 18? Of 12, 15, 20? Of 8, 16, 24?

14. Two men start from the same point and walk around a circular pond. The first makes the circuit in 12 minutes and the second in 15 minutes. How many minutes before they will come together at the starting point?

15. What is the least length of dress goods that can be woven in one piece so that it can be cut without waste into dress patterns of either 6 yards, 10 yards, or 12 yards?

FRACTIONS *Oral*

1. What is a unit? A fraction?

2. In the fraction $\$\frac{3}{4}$, what does each term show?

In a fraction what name is given:

3. To the number below the line? What does it show?

4. To the number above the line? What does it show?

5. Name the fraction which shows what part of the month of September is 1 week; 3 weeks; 1 week 5 days; $2\frac{4}{7}$ weeks.

6. What name is given to a fraction whose value is less than 1? Whose value is 1? Whose value is more than 1?

7. What name is given to a number consisting of a whole number and a fraction?

8. Change to improper fractions:

3 $1\frac{7}{8}$ $3\frac{7}{9}$ $2\frac{9}{10}$ $8\frac{4}{5}$ $6\frac{3}{7}$ $5\frac{5}{6}$ $7\frac{7}{12}$ $9\frac{4}{7}$ $8\frac{3}{11}$

9. Change to mixed numbers:

$\frac{18}{5}$ $\frac{37}{9}$ $\frac{48}{5}$ $\frac{50}{7}$ $\frac{67}{12}$ $\frac{27}{4}$ $\frac{25}{6}$ $\frac{62}{15}$ $\frac{78}{20}$ $\frac{84}{25}$

The form of a fraction may be changed without changing its value. This may be done in two ways:

(1) By changing to higher terms. Thus, $\frac{3}{4} = \frac{6}{8}$.

10. Change to higher terms: $\frac{5}{12}$ $\frac{7}{8}$ $\frac{3}{10}$ $\frac{5}{9}$

11. How is a fraction changed to higher terms?

(2) By changing to lower terms. Thus, $\frac{6}{8} = \frac{3}{4}$.

12. Change to lower terms: $\frac{9}{18}$ $\frac{18}{27}$ $\frac{15}{24}$ $\frac{12}{20}$

13. How is a fraction changed to lower terms?

14. Change to lowest terms: $\frac{14}{16}$ $\frac{10}{15}$ $\frac{18}{24}$ $\frac{24}{30}$ $\frac{30}{36}$

15. When is a fraction in its lowest terms?

16. Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction. What is changed?

WRITTEN EXERCISE

1. Write as fractions with 5 for a denominator:

1 5 8 7 12 15

2. Write five fractions each equal to 1; five equal to $\frac{1}{10}$; five equal to $\frac{3}{4}$.

3. Write $\frac{7}{8}$ in as many forms as possible having two figures in the denominator.

4. Change to 36ths:

1 3 $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{9}$ $\frac{5}{12}$ $\frac{11}{18}$ $\frac{22}{72}$ $\frac{24}{108}$ $\frac{52}{144}$

Change to lowest terms:

5. $\frac{7}{7}$ **6.** $\frac{45}{9}$ **7.** $\frac{27}{63}$ **8.** $\frac{36}{54}$ **9.** $\frac{42}{56}$
10. $\frac{28}{84}$ **11.** $\frac{55}{105}$ **12.** $\frac{72}{108}$ **13.** $\frac{168}{630}$ **14.** $\frac{388}{432}$
15. $\frac{80}{144}$ **16.** $\frac{54}{128}$ **17.** $\frac{120}{576}$ **18.** $\frac{375}{600}$ **19.** $\frac{288}{342}$
20. $\frac{70}{98}$ **21.** $\frac{144}{252}$ **22.** $\frac{85}{120}$ **23.** $\frac{270}{480}$ **24.** $\frac{144}{216}$
25. $\frac{84}{105}$ **26.** $\frac{64}{298}$ **27.** $\frac{84}{144}$ **28.** $\frac{231}{375}$ **29.** $\frac{425}{800}$
30. $\frac{168}{256}$ **31.** $\frac{175}{625}$ **32.** $\frac{135}{252}$ **33.** $\frac{156}{288}$ **34.** $\frac{2000}{2240}$

Change to improper fractions:

35. $12\frac{7}{16}$ **36.** $14\frac{9}{17}$ **37.** $18\frac{3}{11}$ **38.** $19\frac{5}{15}$ **39.** $19\frac{7}{24}$
40. $18\frac{5}{9}$ **41.** $23\frac{5}{16}$ **42.** $25\frac{11}{16}$ **43.** $37\frac{3}{20}$ **44.** $42\frac{7}{25}$
45. $39\frac{12}{19}$ **46.** $58\frac{17}{26}$ **47.** $48\frac{16}{33}$ **48.** $53\frac{7}{32}$ **49.** $67\frac{18}{37}$
50. $73\frac{27}{40}$ **51.** $85\frac{13}{50}$ **52.** $79\frac{17}{48}$ **53.** $87\frac{59}{60}$ **54.** $95\frac{68}{75}$

Change to mixed numbers:

55. $\frac{112}{17}$ **56.** $\frac{182}{35}$ **57.** $\frac{214}{29}$ **58.** $\frac{300}{43}$ **59.** $\frac{423}{57}$
60. $\frac{193}{18}$ **61.** $\frac{207}{24}$ **62.** $\frac{247}{33}$ **63.** $\frac{253}{37}$ **64.** $\frac{521}{45}$
65. $\frac{473}{50}$ **66.** $\frac{385}{62}$ **67.** $\frac{477}{58}$ **68.** $\frac{649}{74}$ **69.** $\frac{377}{60}$
70. $\frac{782}{85}$ **71.** $\frac{649}{88}$ **72.** $\frac{712}{79}$ **73.** $\frac{907}{92}$ **74.** $\frac{887}{96}$

ADDITION AND SUBTRACTION

1. $\frac{3}{4} + \frac{3}{5} = ?$ **2.** $\frac{3}{4} - \frac{3}{5} = ?$
$\frac{15}{20} + \frac{12}{20} = ?$ $\frac{15}{20} - \frac{12}{20} = ?$

3. What change must be made in $\frac{3}{4}$ and $\frac{3}{5}$ before finding their sum or difference?

4. What kind of fractions can be added and subtracted?

Add at sight:

5.	**6.**	**7.**	**8.**	**9.**
$5\frac{1}{2}$	$5\frac{1}{4}$	$6\frac{3}{4}$	$2\frac{1}{2}$	$2\frac{1}{4}$
$2\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{3}{4}$	$4\frac{1}{4}$	$3\frac{3}{8}$
$7\frac{1}{2}$	$2\frac{3}{4}$	$8\frac{3}{4}$	$3\frac{1}{4}$	$2\frac{5}{8}$
$3\frac{1}{2}$	$4\frac{1}{2}$	$7\frac{1}{4}$	$8\frac{3}{4}$	$9\frac{1}{4}$

10.	**11.**	**12.**	**13.**	**14.**
$7\frac{1}{2}$	$3\frac{5}{8}$	$9\frac{7}{8}$	$4\frac{5}{6}$	$7\frac{1}{3}$
$3\frac{1}{4}$	$9\frac{1}{8}$	$8\frac{5}{8}$	$3\frac{1}{2}$	$2\frac{1}{2}$
$2\frac{1}{8}$	$1\frac{1}{4}$	$3\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{3}$
$8\frac{1}{2}$	$5\frac{1}{2}$	$2\frac{3}{4}$	$9\frac{5}{6}$	$4\frac{2}{3}$

15.	**16.**	**17.**	**18.**	**19.**
$8\frac{1}{2}$	$5\frac{1}{4}$	$6\frac{1}{5}$	$8\frac{4}{5}$	$6\frac{1}{4}$
$3\frac{5}{6}$	$2\frac{1}{3}$	$7\frac{1}{2}$	$3\frac{1}{3}$	$5\frac{1}{8}$
$7\frac{1}{6}$	$7\frac{5}{12}$	$9\frac{1}{10}$	$9\frac{1}{5}$	$9\frac{1}{16}$
$9\frac{1}{3}$	$9\frac{2}{3}$	$8\frac{4}{5}$	$5\frac{2}{3}$	$8\frac{1}{2}$

Subtract at sight:

20.	**21.**	**22.**	**23.**	**24.**
$8\frac{1}{2}$	$7\frac{1}{2}$	$4\frac{3}{4}$	$5\frac{2}{3}$	$9\frac{5}{8}$
$2\frac{1}{4}$	$3\frac{1}{3}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$4\frac{1}{2}$

25.	**26.**	**27.**	**28.**	**29.**
$7\frac{1}{2}$	$9\frac{1}{10}$	$6\frac{1}{2}$	$8\frac{1}{2}$	$5\frac{1}{2}$
$3\frac{1}{8}$	$6\frac{1}{5}$	$4\frac{2}{5}$	$3\frac{1}{10}$	$2\frac{1}{6}$

30.	**31.**	**32.**	**33.**	**34.**
$4\frac{5}{6}$	$6\frac{2}{3}$	$5\frac{1}{2}$	$9\frac{1}{3}$	$7\frac{2}{9}$
$1\frac{1}{4}$	$4\frac{1}{4}$	$3\frac{5}{16}$	$3\frac{1}{9}$	$2\frac{2}{3}$

Solve at sight:

35. $\frac{3}{8}+\frac{5}{6}$ **36.** $\frac{4}{5}-\frac{3}{4}$ **37.** $\frac{6}{7}+\frac{4}{21}$ **38.** $\frac{7}{8}-\frac{3}{10}$

39. $\frac{5}{6}+\frac{7}{24}$ **40.** $\frac{7}{10}-\frac{5}{12}$ **41.** $\frac{7}{10}+\frac{4}{15}$ **42.** $\frac{5}{6}-\frac{4}{15}$

43. $\frac{5}{11}+\frac{3}{4}$ **44.** $\frac{8}{9}-\frac{5}{6}$ **45.** $\frac{3}{8}+\frac{7}{12}$ **46.** $\frac{7}{9}-\frac{7}{27}$

47. $\frac{7}{12}+\frac{11}{36}$ **48.** $\frac{8}{15}-\frac{3}{20}$ **49.** $\frac{5}{9}+\frac{8}{15}$ **50.** $\frac{11}{15}-\frac{4}{25}$

51. $\frac{9}{10}+\frac{14}{25}$ **52.** $\frac{11}{12}-\frac{5}{9}$ **53.** $\frac{7}{16}+\frac{1}{3}$ **54.** $\frac{3}{4}-\frac{2}{15}$

WRITTEN EXERCISES

1. Add $\frac{13}{15}$ and $\frac{11}{24}$.

$$\begin{array}{r|rr} 3 & 15 & 24 \\ \hline & 5 & 8 \end{array}$$

$3 \times 5 \times 8 = 120$

$\underline{\text{l. c. d.} = 120}$

$\frac{13}{15} = \frac{104}{120}$

$\frac{11}{24} = \underline{\frac{55}{120}}$

$\frac{159}{120} = 1\frac{39}{120} = 1\frac{13}{40}$

Since the least common denominator is not readily seen, we find the least common multiple of the denominators.

2. $\frac{5}{8} + \frac{7}{12} + \frac{9}{16}$

3. $\frac{2}{15} + \frac{3}{10} + \frac{5}{6}$

4. $\frac{8}{9} + \frac{26}{27} + \frac{2}{3}$

5. $\frac{3}{14} + \frac{11}{42} + \frac{5}{14}$

6. $\frac{5}{6} + \frac{4}{21} + \frac{9}{14}$

7. $\frac{7}{9} + \frac{5}{18} + \frac{7}{12}$

8. $\frac{9}{10} + \frac{14}{15} + \frac{19}{20}$

9. $\frac{7}{12} + \frac{7}{15} + \frac{7}{10}$

10. $\frac{11}{12} + \frac{8}{9} + \frac{17}{18}$

11. $\frac{11}{60} + \frac{7}{12} + \frac{9}{20}$

12. $\frac{3}{14} + \frac{7}{8} + \frac{5}{7}$

13. $\frac{17}{18} + \frac{5}{12} + \frac{17}{24}$

14. $\frac{7}{16} + \frac{17}{28} + \frac{7}{12} + \frac{1}{24}$

15. $\frac{4}{5} + \frac{3}{4} + \frac{7}{20} + \frac{8}{15}$

16. $\frac{3}{7} + \frac{5}{8} + \frac{1}{2} + \frac{3}{4}$

17. $\frac{8}{25} + \frac{2}{3} + \frac{4}{15} + \frac{3}{5}$

18. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$

19. $\frac{3}{5} + \frac{7}{10} + \frac{11}{15} + \frac{13}{20} + \frac{5}{12}$

20. $\frac{1}{7} + \frac{1}{8} + \frac{3}{4} + \frac{1}{2} + \frac{5}{14}$

21. $\frac{5}{6} + \frac{2}{7} + \frac{1}{14} + \frac{2}{3} + \frac{5}{21}$

22. Take $12\frac{13}{18}$ from $16\frac{17}{30}$.

$\underline{\text{l. c. d.} = 90}$

$16\frac{17}{30} = 16\frac{51}{90} = 15\frac{141}{90}$

$12\frac{13}{18} = 12\frac{65}{90} = \underline{12\frac{65}{90}}$

$3\frac{76}{90} = 3\frac{38}{45}$

Since we cannot take $\frac{65}{90}$ from $\frac{51}{90}$, we take 1 from 16, express it as $\frac{90}{90}$, and add it to $\frac{51}{90}$, making $\frac{141}{90}$.

Find the value of:

23. $7\frac{3}{8} + 4\frac{7}{12}$

24. $8\frac{5}{6} - 3\frac{7}{8}$

25. $9\frac{2}{3} + 7\frac{5}{8}$

26. $6\frac{3}{7} - 2\frac{5}{6}$

27. $8\frac{3}{4} + 8\frac{4}{5}$

28. $9\frac{1}{12} - 4\frac{3}{7}$

29. $4\frac{1}{6} + 8\frac{4}{5}$

30. $7\frac{3}{10} - 5\frac{2}{3}$

31. $5\frac{1}{6} + 5\frac{1}{9}$

32. $8\frac{3}{7} - 2\frac{5}{8}$

33. $7\frac{8}{9} + 5\frac{6}{7}$

34. $4\frac{5}{9} - 1\frac{7}{12}$

35. $12\frac{4}{9} + 7\frac{8}{15}$

36. $10\frac{4}{15} - 5\frac{7}{20}$

37. $16\frac{9}{25} + 8\frac{7}{10}$

38. $19\frac{5}{12} - 8\frac{11}{16}$ **39.** $18\frac{14}{15} + 12\frac{5}{18}$ **40.** $20\frac{17}{25} - 12\frac{13}{15}$

41. $25\frac{11}{24} + 16\frac{17}{18}$ **42.** $15\frac{1}{12} - 11\frac{7}{15}$ **43.** $28\frac{7}{20} + 15\frac{13}{18}$

44. $25\frac{10}{27} - 18\frac{8}{15}$ **45.** $36\frac{4}{35} + 42\frac{7}{10}$ **46.** $50\frac{9}{24} - 24\frac{7}{30}$

MULTIPLICATION OF FRACTIONS

Give products at sight:

1. $5 \times \frac{3}{4}$ **2.** $3 \times \frac{7}{8}$ **3.** $9 \times \frac{4}{5}$ **4.** $7 \times \frac{3}{10}$

5. $8 \times 1\frac{2}{3}$ **6.** $7 \times 4\frac{1}{7}$ **7.** $8 \times 2\frac{2}{3}$ **8.** $10 \times 3\frac{3}{5}$

9. $\frac{8}{9} \times 72$ **10.** $\frac{5}{8} \times 10$ **11.** $\frac{7}{9} \times 54$ **12.** $\frac{2}{3} \times 11$

13. $3\frac{1}{4} \times 80$ **14.** $\frac{7}{12} \times 9$ **15.** $\frac{6}{7} \times 84$ **16.** $\frac{2}{3} \times 16$

17. $\frac{3}{4} \times \frac{8}{15}$ **18.** $\frac{3}{5} \times \frac{4}{27}$ **19.** $\frac{7}{8} \times \frac{4}{5}$ **20.** $\frac{2}{9} \times \frac{5}{7}$

21. $\frac{5}{7} \times \frac{14}{15}$ **22.** $\frac{9}{10} \times \frac{2}{13}$ **23.** $\frac{3}{11} \times \frac{5}{18}$ **24.** $\frac{4}{5} \times \frac{11}{13}$

Solve: *Written*

25. $15 \times \frac{7}{27}$ **26.** $21 \times \frac{5}{8}$ **27.** $8 \times \frac{31}{44}$ **28.** $9 \times \frac{14}{15}$

29. $7 \times 5\frac{9}{10}$ **30.** $36 \times 3\frac{1}{6}$ **31.** $25 \times 5\frac{4}{5}$ **32.** $6 \times 12\frac{2}{5}$

33. $\frac{15}{16} \times 80$ **34.** $\frac{9}{25} \times 95$ **35.** $\frac{14}{23} \times 5$ **36.** $\frac{71}{96} \times 15$

37. $3\frac{7}{9} \times 52$ **38.** $4\frac{4}{15} \times 8$ **39.** $5\frac{3}{10} \times 25$ **40.** $15\frac{5}{6} \times 28$

41. $\frac{17}{24} \times \frac{8}{51}$ **42.** $\frac{9}{11} \times \frac{121}{431}$ **43.** $\frac{27}{35} \times \frac{6}{19}$ **44.** $\frac{3}{16} \times \frac{80}{93}$

45. $\frac{152}{287} \times \frac{7}{8}$ **46.** $\frac{11}{16} \times \frac{18}{77}$ **47.** $\frac{25}{144} \times \frac{48}{125}$ **48.** $\frac{28}{165} \times \frac{35}{84}$

DIVISION

1. Divide $\frac{4}{7}$ by $\frac{2}{3}$.

Change to like fractions, and divide the numerator of the dividend by the numerator of the divisor.

$$\frac{4}{7} \div \frac{2}{3} = \frac{12}{21} \div \frac{14}{21} = 12 \div 14 = \frac{12}{14} = \frac{6}{7}$$

Or, for convenience, invert the divisor and multiply, canceling when possible.

$$\frac{4}{7} \div \frac{2}{3} = \frac{\overset{2}{\cancel{4}}}{7} \times \frac{3}{\cancel{2}} = \frac{6}{7}$$

Solve at sight:

2. $3 \div \frac{1}{4}$ **3.** $6 \div \frac{2}{3}$ **4.** $8 \div \frac{4}{5}$ **5.** $12 \div \frac{6}{7}$
6. $5 \div \frac{1}{5}$ **7.** $7 \div \frac{14}{15}$ **8.** $9 \div \frac{3}{11}$ **9.** $15 \div \frac{5}{6}$
10. $\frac{2}{3} \div 4$ **11.** $\frac{3}{4} \div 6$ **12.** $\frac{5}{8} \div 10$ **13.** $\frac{4}{9} \div 12$
14. $\frac{3}{8} \div 6$ **15.** $\frac{9}{10} \div 3$ **16.** $\frac{4}{9} \div 6$ **17.** $\frac{1}{6} \div 5$
18. $\frac{5}{8} \div \frac{1}{5}$ **19.** $\frac{8}{9} \div \frac{1}{3}$ **20.** $\frac{5}{6} \div \frac{2}{3}$ **21.** $\frac{3}{4} \div \frac{2}{3}$
22. $\frac{7}{8} \div \frac{7}{4}$ **23.** $\frac{5}{6} \div \frac{1}{12}$ **24.** $\frac{3}{10} \div \frac{3}{4}$ **25.** $\frac{4}{9} \div \frac{2}{3}$

Solve: *Written*

26. $8 \div 3\frac{1}{5}$ **27.** $12 \div 5\frac{4}{5}$ **28.** $17 \div 4\frac{6}{7}$ **29.** $20 \div 8\frac{3}{4}$
30. $24 \div 6\frac{4}{7}$ **31.** $25 \div 4\frac{3}{8}$ **32.** $36 \div 3\frac{3}{7}$ **33.** $42 \div 5\frac{5}{6}$
34. $50 \div 8\frac{2}{3}$ **35.** $72 \div 7\frac{1}{5}$ **36.** $84 \div 9\frac{3}{5}$ **37.** $96 \div 8\frac{8}{9}$
38. $12\frac{9}{16} \div 3$ **39.** $15\frac{3}{7} \div 2$ **40.** $8\frac{3}{10} \div 4$ **41.** $16\frac{2}{3} \div 7$
42. $20\frac{1}{4} \div 8$ **43.** $32\frac{1}{5} \div 9$ **44.** $24\frac{3}{7} \div 5$ **45.** $12\frac{9}{20} \div 3$
46. $44\frac{4}{9} \div 9$ **47.** $67\frac{2}{3} \div 7$ **48.** $42\frac{3}{7} \div 8$ **49.** $65\frac{5}{8} \div 4$
50. $80\frac{5}{16} \div 6$ **51.** $56\frac{3}{11} \div 5$ **52.** $29\frac{5}{12} \div 7$ **53.** $29\frac{2}{9} \div 8$
54. $30\frac{3}{10} \div 9$ **55.** $90\frac{7}{8} \div 7$ **56.** $60\frac{2}{3} \div 5$ **57.** $66\frac{2}{3} \div 6$
58. $67\frac{1}{2} \div 8$ **59.** $62\frac{1}{2} \div 9$ **60.** $83\frac{1}{3} \div 7$ **61.** $125\frac{5}{8} \div 12$
62. $4\frac{1}{4} \div 6\frac{4}{5}$ **63.** $3\frac{6}{7} \div 3\frac{3}{8}$ **64.** $9\frac{3}{5} \div 3\frac{5}{9}$ **65.** $4\frac{1}{11} \div 6\frac{3}{7}$
66. $12\frac{3}{8} \div 15\frac{5}{8}$ **67.** $16\frac{2}{3} \div 18\frac{3}{4}$ **68.** $37\frac{1}{2} \div 13\frac{1}{3}$ **69.** $33\frac{1}{3} \div 14\frac{2}{7}$
70. $5\frac{6}{7} \div 6\frac{5}{7}$ **71.** $4\frac{4}{9} \div 6\frac{2}{3}$ **72.** $5\frac{5}{8} \div 8\frac{2}{5}$ **73.** $7\frac{9}{16} \div 2\frac{5}{24}$
74. $18\frac{3}{4} \div 12\frac{7}{9}$ **75.** $14\frac{2}{5} \div 10\frac{2}{7}$ **76.** $42\frac{6}{7} \div 22\frac{2}{9}$ **77.** $62\frac{1}{2} \div 83\frac{1}{3}$

COMPLEX FRACTIONS

Such expressions as $\frac{2}{3\frac{1}{2}}$, $\frac{4\frac{1}{5}}{3}$, $\frac{3\frac{1}{5}}{2\frac{1}{3}}$, are *complex fractions*.

They are simplified by performing the operation indicated.

1. Find the value of $\frac{3\frac{3}{4}}{1\frac{2}{3}}$.

$$(1)\quad \frac{3\frac{3}{4}}{1\frac{2}{3}} = 3\tfrac{3}{4} \div 1\tfrac{2}{3} = \frac{\overset{3}{\cancel{15}}}{4} \times \frac{3}{\cancel{5}} = \frac{9}{4} = 2\tfrac{1}{4}$$

Since multiplying both terms of a fraction by the same number does not change the value of the fraction, we may simplify by multiplying both terms by the least common multiple of the denominators.

$$(2)\quad \frac{3\frac{3}{4} \times 12 = 45}{1\frac{2}{3} \times 12 = 20} = \frac{9}{4} = 2\tfrac{1}{4}$$

Solve:

2. $\frac{3\frac{1}{3}}{1\frac{2}{3}}$ **3.** $\frac{2\frac{1}{2}}{1\frac{1}{2}}$ **4.** $\frac{1\frac{3}{5}}{5\frac{1}{3}}$ **5.** $\frac{5\frac{1}{3}}{2\frac{1}{4}}$ **6.** $\frac{15\frac{3}{5}}{12\frac{2}{3}}$

7. $\frac{7\frac{1}{2}}{6\frac{2}{3}}$ **8.** $\frac{2\frac{2}{5}}{3\frac{3}{4}}$ **9.** $\frac{2\frac{1}{4}}{3\frac{1}{8}}$ **10.** $\frac{2\frac{5}{8}}{2\frac{2}{3}}$ **11.** $\frac{8\frac{1}{8}}{3\frac{2}{5}}$

12. $\frac{20\frac{1}{4}}{15\frac{3}{4}}$ **13.** $\frac{6\frac{2}{3}}{7\frac{5}{6}}$ **14.** $\frac{14\frac{2}{5}}{4\frac{3}{8}}$ **15.** $\frac{18\frac{3}{4}}{16\frac{2}{3}}$ **16.** $\frac{5\frac{5}{7}}{13\frac{1}{3}}$

Solve, first simplifying the parts in parentheses:

17. $(\frac{2}{3} + \frac{4}{5}) \div (\frac{1}{5} + \frac{1}{6})$ **18.** $(\frac{7}{10} + \frac{15}{21}) \div (\frac{7}{8} + \frac{3}{10})$

19. $(\frac{1}{2} + \frac{2}{3}) \div (\frac{4}{5} - \frac{1}{2})$ **20.** $(\frac{7}{20} \times 6\frac{2}{3}) + (1\frac{1}{4} \times 2\frac{1}{2})$

21. $(\frac{3}{4} \div 3\frac{1}{5}) \times (\frac{4}{5} - \frac{2}{3})$ **22.** $(3\frac{5}{8} + 2\frac{7}{8}) \times (3\frac{5}{8} - 2\frac{1}{4})$

23. $(2\frac{3}{8} + 1\frac{1}{5}) - (6\frac{1}{4} - 5\frac{1}{8})$ **24.** $(5\frac{4}{9} - 3\frac{1}{6}) \div (3\frac{3}{8} \times 1\frac{7}{9})$

25. $(4\frac{2}{7} \times 4\frac{1}{5}) - (8\frac{3}{4} - 4\frac{3}{10})$ **26.** $(2\frac{3}{8} + 1\frac{1}{5}) - (\frac{17}{20} \div 3\frac{2}{5})$

MISCELLANEOUS PROBLEMS

1. Mr. Bryant sold three piles of wood containing $8\frac{3}{4}$ cords, $4\frac{9}{16}$ cords, and $7\frac{5}{8}$ cords, respectively. How many cords in all?

2. Eight drinking fountains were placed in a school building at an average expense of \$$6\frac{4}{5}$. How much did they cost?

3. A post $8\frac{1}{2}$ feet long is set $2\frac{2}{3}$ feet in the ground. What is the length above ground?

4. A special electric car was hired by a party for \$$22\frac{3}{4}$. If each person paid \$$\frac{7}{20}$, how many went on the trip?

5. Mrs. Appleton bought a piece of beef weighing $7\frac{3}{4}$ pounds. One third of the weight was lost in roasting. What was the weight of the roast when ready for the table?

6. Wallace bought a sled for \$$\frac{4}{5}$, and a knife for $\frac{9}{16}$ as much. How much did he pay for both?

7. At \$$8\frac{4}{5}$ a cord, what is the value of a pile of wood containing $1\frac{7}{8}$ cords?

8. From a board $13\frac{1}{12}$ feet long there were cut 2 pieces, each $6\frac{3}{4}$ feet long. What was the waste?

9. How much must a carpenter be paid for 38 hours' work if he receives \$3.50 per day of 8 hours?

10. If a merchant pays \$$\frac{7}{25}$ apiece for melons and sells them at \$$\frac{2}{5}$ apiece, what is his gain?

11. A ton of lime for fertilizing cost \$$6\frac{3}{4}$. It cost $\frac{2}{9}$ as much to cart it from the freight depot to the farm. What was the total cost?

12. A merchant sold a piece of cloth containing $3\frac{5}{8}$ yards for \$$10\frac{7}{8}$. At the same rate per yard, he sold another piece for \$$17\frac{1}{4}$. How many yards in the second piece?

13. A merchant sells $16\frac{2}{3}$ dozen pairs of shears at \$$\frac{3}{4}$ per pair. How much does he receive for them?

14. A cargo of lumber valued at $\$6500$ was lost at sea. Mr. James owned $\frac{3}{13}$ of it. What was his loss?

15. At the village store, Mrs. Linwood exchanged $8\frac{1}{3}$ dozen eggs at 30 cents a dozen for $3\frac{1}{8}$ yards of cloth. What was the price of the cloth a yard?

16. The price of a railroad ticket between two places is 90 cents. How much will it be if increased $\frac{1}{5}$? How much if increased $\frac{1}{5}$ of a dollar?

17. An electric railway is $16\frac{5}{8}$ miles long. If it followed the highway, its length would be $22\frac{5}{8}$ miles. How many miles are saved by not following the highway?

18. At $\$\frac{2}{5}$ per gallon, what is the value of 27 barrels of machine oil, each containing 48 gallons?

19. Two fifths of Mr. Johnson's farm is in pasture, $\frac{1}{3}$ in hay and grain, and the rest in other crops. What part in other crops?

20. Three boys own a boat. Robert owns $\frac{2}{5}$ of it, Raymond owns $\frac{1}{3}$ of it. What part of it does George own? If they sell the boat for $\$30$, how much ought each to receive?

21. At $\$27\frac{1}{2}$ a ton, what is the cost of $62\frac{1}{2}$ tons of steel rails?

22. A farmer paid $\$35\frac{3}{4}$ for plowing $8\frac{1}{8}$ acres of land. How much per acre?

23. The perimeter of a triangular lot of land is $65\frac{5}{9}$ rods. Two of the sides are $18\frac{2}{3}$ rods and $25\frac{1}{6}$ rods, respectively. What is the length of the third side?

24. Robert sells, for 5 cents a copy, a weekly magazine which he buys for $3\frac{1}{2}$ cents a copy. He makes 75 cents a week. How many customers has he?

25. What is the gain when 1260 bushels of wheat are bought at 87½ cents a bushel and sold at 93¾ cents a bushel?

26. A farmer paid $120 for a horse and sold it at a gain of ⅛ of the cost. What was the gain? What was the selling price?

27. When hats bought at the rate of 5 for $3 are sold at the rate of 3 for $5, what is the profit on 5 cases of 4 dozen each?

28. When hay was worth $16 a ton, Mr. Parker sold 1¾ tons to Mr. Hatch, receiving in payment 48 bushels of oats and $6.40 in money. What was the value of the oats per bushel?

29. Oscar lives 2¾ miles from Paul. When Oscar has walked ⅝ of the distance to Paul's home, how far has Paul to walk to meet him?

30. Two men harvested a farmer's apples for $120. If one man did ⅗ of the work, how much ought each to get?

31. At another time these same men earned $120 in cutting timber in the wood lot, one cutting ⅗ as much as the other. How much did each earn?

32. A house was sold at a profit of $861. This was ⅐ more than it cost. What was the cost? The selling price?

33. A farmer bought a two-horse sulky plow for $33.75, and by renting it at 75 cents a day received enough money to pay ⅖ of the cost. How many days did he rent it?

34. A building lot containing 10,800 square feet was bought for $⅛ per square foot and sold for $⅙ per square foot. What was the gain?

35. If Mr. Hawley owns $\frac{3}{20}$ of a mine and sells $\frac{5}{9}$ of his share, what part of the mine does he still own?

36. When $\frac{2}{3}$ of a ship is sold for $3780, what is the ship worth?

37. A barge containing 1625 tons of coal must be unloaded in 5 days. If the bucket used holds $\frac{3}{4}$ of a ton, how many bucketfuls must be taken out each day?

38. A fruit merchant buys apples at $\$1\frac{2}{5}$ per barrel and sells at $\$1\frac{3}{4}$ per barrel. If the expense of handling is $\$\frac{1}{10}$ per barrel, what is his profit in buying and selling 640 barrels?

39. A workman in a shoe manufactory receives 32 cents an hour for a day's work of 8 hours. When he works overtime, each extra hour is reckoned as $1\frac{1}{2}$ hours. How much does he earn in a week when he works as follows: Monday 8 hours, Tuesday 10 hours, Wednesday $9\frac{1}{2}$ hours, Thursday 9 hours, Friday $10\frac{1}{2}$ hours, and Saturday 11 hours?

40. A tub of butter weighed 56 pounds. The grocer sold $\frac{1}{2}$ of it to one customer and $\frac{1}{2}$ the remainder to another customer. How many pounds were left?

41. After using $3\frac{5}{8}$ yards of cloth at one time, and $\frac{2}{3}$ as much at another, $7\frac{3}{4}$ yards were left. How many yards at first?

42. One mason can lay a wall in 12 days; another mason can lay it in 15 days. If the wages of the first are $4 per day, what ought to be the daily wages of the second?

43. A gang of men can build a piece of road in $2\frac{2}{3}$ weeks, working 10 hours a day. How long will it take if they work 8 hours a day?

44. James and Arthur, working together, can hoe a field of corn in 6 days. How long will it take either alone?

45. The same boys can each hoe another field in 8 days. How long will it take them working together?

46. Three men, working together, can clapboard a house in 4 days. How long will it take one man alone? Two men?

47. One man can cultivate a field of beets in 18 hours. How long will it take 2 men? How long will it take 3 men?

48. Mr. Cross can plant a field to potatoes in 4 days; his son Albert can plant it in 12 days. What part of the work can Mr. Cross do in 1 day? Albert? Mr. Cross and Albert together? How many days will it take both together to plant the field?

49. One man can do a piece of work in 6 days and another man can do the same work in 8 days. How long will it take both together to do the work?

Together in one day they can do $\frac{1}{6} + \frac{1}{8}$, or $\frac{7}{24}$ of the work. The fraction $\frac{7}{24}$ tells that if the work is divided into 24 equal parts, both men together can do 7 of these parts in one day. Therefore, it will take as many days for both together to do the work as 7 is contained times in 24. *Ans.*, $3\frac{3}{7}$ days.

50. A gang of men can build a sewer in 6 weeks. Another gang can build it in 4 weeks. How long will it take both gangs together?

51. An oil reservoir in Pennsylvania has three outlets. The first can empty it in 10 hours, the second in 12 hours, and the third in 15 hours. How long will it take the first and second together? The first and third?

The second and third? Find the shortest time in which the reservoir can be emptied.

52. Emma and Clara can do the week's ironing in 2 hours. Emma alone in 6 hours. In what time can Clara do it alone?

REVIEW EXERCISE

	A		*B*		*C*		*D*	
1.	$\frac{10}{12}$	$\frac{16}{48}$	$\frac{19}{38}$	$\frac{75}{100}$	$\frac{60}{75}$	$\frac{10}{100}$	$\frac{28}{63}$	$\frac{28}{56}$
2.	$\frac{6}{9}$	$\frac{27}{54}$	$\frac{32}{36}$	$\frac{15}{40}$	$\frac{49}{84}$	$\frac{40}{48}$	$\frac{36}{54}$	$\frac{25}{35}$
3.	$\frac{18}{27}$	$\frac{15}{35}$	$\frac{28}{49}$	$\frac{14}{18}$	$\frac{63}{72}$	$\frac{15}{40}$	$\frac{36}{64}$	$\frac{27}{72}$
4.	$\frac{27}{30}$	$\frac{15}{40}$	$\frac{28}{32}$	$\frac{15}{25}$	$\frac{9}{24}$	$\frac{27}{36}$	$\frac{49}{56}$	$\frac{35}{40}$
5.	$\frac{12}{18}$	$\frac{6}{9}$	$\frac{21}{35}$	$\frac{25}{30}$	$\frac{21}{24}$	$\frac{14}{14}$	$\frac{21}{27}$	$\frac{36}{42}$
6.	$\frac{24}{72}$	$\frac{3}{15}$	$\frac{2}{15}$	$\frac{8}{24}$	$\frac{24}{36}$	$\frac{25}{45}$	$\frac{12}{18}$	$\frac{40}{72}$
7.	$\frac{18}{60}$	$\frac{7}{35}$	$\frac{9}{15}$	$\frac{15}{18}$	$\frac{24}{48}$	$\frac{51}{60}$	$\frac{10}{12}$	$\frac{15}{25}$
8.	$\frac{27}{45}$	$\frac{15}{40}$	$\frac{12}{18}$	$\frac{14}{21}$	$\frac{6}{18}$	$\frac{3}{60}$	$\frac{12}{16}$	$\frac{20}{35}$
9.	$\frac{96}{144}$	$\frac{24}{84}$	$\frac{18}{27}$	$\frac{27}{30}$	$\frac{27}{45}$	$\frac{21}{35}$	$\frac{33}{44}$	$\frac{63}{72}$
10.	$\frac{27}{84}$	$\frac{12}{15}$	$\frac{22}{33}$	$\frac{44}{55}$	$\frac{25}{30}$	$\frac{45}{60}$	$\frac{17}{34}$	$\frac{8}{40}$

1. Change the fractions in each couplet to lowest terms, and then find the sum and the difference of each couplet.

2. Find the product of each couplet.

3. Divide the first term of each couplet by the second term.

4. Tell at sight what must be added to each of these numbers to make 100:

$2\frac{1}{2}$ $3\frac{1}{3}$ $4\frac{1}{6}$ $6\frac{1}{4}$ $6\frac{2}{3}$ $8\frac{1}{3}$ $12\frac{1}{2}$
$16\frac{2}{3}$ $33\frac{1}{3}$ $37\frac{1}{2}$ $62\frac{1}{2}$ $66\frac{2}{3}$ $83\frac{1}{3}$ $87\frac{1}{2}$

5. Find the sum and the difference of each of the above numbers and 50; 25; 75; $16\frac{2}{3}$; $33\frac{1}{3}$; $66\frac{2}{3}$; $83\frac{1}{3}$; $12\frac{1}{2}$; $37\frac{1}{2}$; $62\frac{1}{2}$; $87\frac{1}{2}$.

DECIMALS

1. Is a decimal a whole number or a fraction?

A power of a number is the product obtained by using that number a specified number of times as a factor.

Thus, the second power of 10 is 10 × 10, or 100.

2. What is the third power of ten?

3. What power of 10 is represented by 10^4? What is the fourth power of 10?

4. What must the denominator of a decimal be?

5. Write as decimals: $\frac{3}{10}$ $\frac{3}{100}$ $\frac{3}{1000}$ $\frac{3}{10000}$ $\frac{3}{1000000}$

6. When a common fraction whose denominator is 10 or some power of 10 is expressed as a decimal, how is the number of decimal places determined?

7. Write as common fractions: .9 .09 .009 .0009 .00009

8. When a decimal is expressed as a common fraction, how is the denominator determined?

DECIMALS: NOTATION AND NUMERATION

	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
.	3	2	8	6	5	7

Read:

1. .007 **2.** .0035 **3.** .0105 **4.** .12067

5. .0102 **6.** .2008 **7.** .013246 **8.** .687259

9. 35.96 **10.** 8171.6 **11.** 34.0207 **12.** 3.29018

13. 4.067 **14.** 810.05 **15.** 80070.8 **16.** 49230.7

17. 643.7 **18.** 3.4907 **19.** 640.048 **20.** 500.005

21. What is a mixed decimal?

Write in figures:

22. Thirty-seven hundredths.

23. Nine thousandths.

24. Forty-two ten-thousandths.

25. Seven hundred sixteen hundred-thousandths.

26. One hundred eighty and sixty-nine hundredths.

27. Five thousand forty-eight and six thousandths.

28. Seventy and sixty-one hundred thousandths.

29. Four hundred four thousandths.

30. Four hundred and four thousandths.

31. One hundred thousand one hundred and one hundredth.

DECIMAL AND FRACTIONAL EQUIVALENTS

Write as common fractions in their lowest terms:

1. .35 **2.** .65 **3.** .85 **4.** .32

5. .48 **6.** .72 **7.** .56 **8.** .96

9. .64 **10.** .28 **11.** .84 **12.** .36

13. .125 **14.** .375 **15.** .225 **16.** .875

17. .625 **18.** .025 **19.** .075 **20.** .015

21. .0025 **22.** .0375 **23.** .0625 **24.** .0875

25. .0125 **26.** .425 **27.** .105 **28.** .275

Tell at sight the decimal equivalents of:

29. $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{1}{25}$ $\frac{4}{25}$ $\frac{9}{25}$

30. $\frac{1}{10}$ $\frac{3}{10}$ $\frac{7}{10}$ $\frac{9}{10}$ $\frac{1}{20}$ $\frac{3}{20}$ $\frac{11}{20}$ $\frac{1}{50}$ $\frac{3}{50}$ $\frac{9}{50}$

31. Change $\frac{7}{8}$ to a decimal.

$$\frac{7}{8} = 8)\underline{7.000}$$
$$.875$$

32. Change $\frac{2}{3}$ to a decimal.

$$\frac{2}{3} = 3)\underline{2.000}$$
$$.666\frac{2}{3}, \text{ or } .666^{+}$$

Change to decimals:

33. $\frac{1}{3}$ **34.** $\frac{1}{6}$ **35.** $\frac{5}{6}$ **36.** $\frac{5}{7}$ **37.** $\frac{1}{8}$

38. $\frac{3}{8}$ **39.** $\frac{5}{8}$ **40.** $\frac{7}{8}$ **41.** $\frac{1}{9}$ **42.** $\frac{1}{11}$

43. $\frac{5}{12}$ **44.** $\frac{1}{15}$ **45.** $\frac{1}{24}$ **46.** $\frac{1}{30}$ **47.** $\frac{5}{32}$

48. $\frac{1}{40}$ **49.** $\frac{1}{80}$ **50.** $\frac{63}{64}$ **51.** $\frac{75}{128}$ **52.** $\frac{103}{256}$

.18, .625, .004 are pure decimals; $.16\frac{2}{3}$, $.01\frac{1}{7}$, $.006\frac{2}{9}$ are complex decimals.

53. What is a pure decimal? A complex decimal?

Since the denominator of a decimal is 10 or some power of 10, the only common fractions that can be expressed as pure decimals are those whose denominators, when the fractions are in their lowest terms, contain only the factors 2 or 5.

54. Name five fractions that can be changed to pure decimals; five that cannot be changed to pure decimals.

$.6\frac{1}{2}$ means 6 tenths and $\frac{1}{2}$ of a tenth, and may be written .65.

Write as pure decimals:

55. $.2\frac{1}{5}$ **56.** $.4\frac{1}{4}$ **57.** $.3\frac{2}{5}$ **58.** $.1\frac{3}{4}$ **59.** $.7\frac{3}{5}$

60. $.8\frac{4}{5}$ **61.** $.2\frac{1}{8}$ **62.** $.4\frac{3}{8}$ **63.** $.6\frac{5}{8}$ **64.** $.3\frac{7}{8}$

DECIMALS: ADDITION

Write in decimal form and find the sum of:

1. $3\frac{67}{100}$, $45\frac{891}{10000}$, $12\frac{1}{10}$, $26\frac{37}{1000}$, $5\frac{8}{100}$.
2. $24\frac{7}{10}$, $\frac{16}{1000}$, $3\frac{7}{100000}$, $37\frac{94}{100}$, $100\frac{1}{100}$.
3. $\frac{9}{10}$, $\frac{9}{100}$, $\frac{9}{1000}$, $\frac{9}{10000}$, $\frac{9}{100000}$.
4. $\frac{7}{10}$, $\frac{777}{1000}$, $\frac{7777}{100000}$, $\frac{77}{100}$, $\frac{7777}{10000}$.

Find the sum of:

5. 43.9, 65.4, 1.93, .412.
6. 1.07, 5.002, 37.6, .308.
7. .006, .08, .0007, .9.
8. 4.875, 6.375, 12.08, 17.016.
9. 43.225, 17.906, 10.017, 3.916.
10. .0007, .0165, .904, .8, .217.
11. 156.28, 375.009, 67.803, .0805.
12. .1, 8.03, 7.005, 8.0007, 5.00009.
13. 7 hundredths; 15 thousandths; 65 hundredths; 5 tenths; 8 ten-thousandths.
14. 95 ten-thousandths; 389 thousandths; 265 ten-thousandths.
15. Six hundred and six thousandths; eight hundred and four tenths; seventy and eighty-six ten-thousandths.

16.	17.	18.	19.
.7854	25.256	240.24	371.876
.10842	102.102	37.1	90.84
.98003	6708.00093	1259.008	27.3071
.500004	812.0104	760.00716	19.0008
.70063	73.0016	15.005	1024.01018

DECIMALS: SUBTRACTION

Subtract at sight:

1.	2.4	**2.**	6.8	**3.**	.47	**4.**	.007	**5.**	.0075
	.7		.9		.12		.005		.0025
6.	2.75	**7.**	3.50	**8.**	3.7	**9.**	5.3	**10.**	.02
	1.5		3.25		2.8		.9		.002

11. From 1 take .5; .25; .75; .42; .67.

12. From 1 take .1; .01; .001; .0001.

13. From 4 take 1.25; 1.5; 1.75; 1.05.

14. From .5 take .25; .05; .49; .025.

15. From 10.1 take 10; .1; 1.1; 1.01; 5.5.

16.	12.42	**17.**	7.645	**18.**	6.85	**19.**	.909
	7.09		.409		.57		.0909
20.	.807	**21.**	.073	**22.**	.005	**23.**	.68
	.69		.0085		.0027		.0375

24. From eight and seventy-five hundredths take ninety-two hundredths.

25. From sixty-seven hundredths take seventy-nine thousandths.

26. Find the difference between sixty-two and eight hundredths, and twelve and eighty-six thousandths.

27. Subtract twenty-six thousandths from two and five-tenths.

28. Take ten and one hundredth from one hundred and one tenth.

29. Subtract three hundredths from three hundred.

30. From two hundred two take two hundred two thousandths.

31. By how much is one hundred and one hundredth greater than one hundred one hundredths?

DECIMALS: MULTIPLICATION

1. 34.6 multiplied by 10 =

2. 34.6 multiplied by 100 =

3. 34.6 multiplied by 1000 =

4. 34.6 multiplied by 10000 =

5. 34.6 multiplied by .1 =

6. 34.6 multiplied by .01 =

7. 34.6 multiplied by .001 =

8. 34.6 multiplied by .0001 =

9. Tell how to multiply by 10. By 100. By 1000. By 10,000.

10. Tell how to multiply by .1. By .01. By .001. By .0001.

11. Express 648 dollars as cents; 2750 cents as dollars.

Multiply:

12. $0.2475 by 10; by 100; by 1000; by 10,000.

13. $9625 by .1; by .01; by .001; by .0001.

14. 375 by .1

15. 468.2 by 10

16. 14.76 by 10

17. 3.462 by 100

18. 13.459 by .01

19. 293.08 by .1

20. 7.696 by .001

21. 172.8 by 1000

22. 48,700 by .01

23. 470.06 by .01

24. 13.629 by .1

25. 34,000 by .001

26. $\frac{7}{10} \times \frac{7}{10} = \quad .7 \times .7 =$

27. $\frac{7}{100} \times \frac{7}{10} = \quad .07 \times .7 =$

28. $\frac{7}{100} \times \frac{7}{100} = .07 \times .07 =$

29. How many decimal places in the product of .7 × .7? In the product of .07 × .7? In the product of .07 × .07?

30. Tell how to point off the product in the multiplication of decimals.

Multiply:

31. 3068 by .8

32. 60.5 by 2.7

33. 12.56 by .06

34. 6.05 by 6.02

35. 684.7 by .15

36. 3.72 by 1.1

37. .008 by 6.7

38. 92.43 by 1.8

39. 3.1416 by 75

40. .7854 by 55

41. .0606 by 3.6

42. 37.856 by 1000

43. .093 by .07

44. .008 by .055

45. 376.04 by 1.05

46. 12.75 by .032

47. 3025 by .098

48. .318 by 2.75

49. .03 by 3600

50. .007 by .005

51. 4.0804 by 5.05

52. 462.07 by .08

53. 36.036 by 12.3

54. 378.5 by 1.35

55. .009 by .072

56. .55 by 5.5

57. 66.3 by .064

58. .00808 by 1000

59. 1428 by .139

60. 4.096 by .045

61. .015 by 200.2

62. .1504 by 8.08

63. .505 by 80.8

64. .375 by 4400

65. .0213 by 6.8

66. 3.03 by .0207

67. .0825 by .026
68. 36.84 by .0105
69. 6728 by .0005
70. .0064 by 29
71. .00086 by 295
72. 27.004 by .025
73. 1.032 by 8.8
74. 63360 by .075
75. .0075 by .044

76. Multiply $2.40 by 500.

$2.40 multiplied by 100 = $240; $240 multiplied by 5 = $1200.

77. Multiply $2.40 by .05.

$2.40 multiplied by .01 = $0.024; $0.024 multiplied by 5 = $0.12.

Multiply:

78. $26 by 200
79. $18.60 by 20
80. $16.50 by 30
81. $36 by .5
82. $22.25 by .2
83. $12.75 by .4
84. $32.50 by 300
85. $12.25 by 400
86. $1.25 by 600
87. $75 by .6
88. $3.50 by .02
89. $42.50 by .08
90. $16 by .3
91. $810 by .07
92. $3600 by .002
93. $4800 by .005

DECIMALS: DIVISION

Test these divisions:

1. $\begin{array}{r} 7\underline{).56} \\ .08 \end{array}$ **2.** $\begin{array}{r} 4\underline{).032} \\ .008 \end{array}$ **3.** $\begin{array}{r} 5\underline{).4} \\ .08 \end{array}$ **4.** $\begin{array}{r} 6\underline{).054} \\ .009 \end{array}$ **5.** $\begin{array}{r} 4\underline{).02} \\ .005 \end{array}$

6. Find the quotient of (1) .64 ÷ .2; (2) .056 ÷ .07; (3) 824 ÷ 2.06.

$$(1)\ .2\underline{).64} = \begin{array}{r} 2\underline{)6.4} \\ 3.2 \end{array}$$

$$(2)\ .07\underline{).056} = \begin{array}{r} 7\underline{)5.6} \\ .8 \end{array}$$

$$(3)\ 2.06\overline{)824} = \begin{array}{r} 400 \\ 206\overline{)82400} \end{array}$$

7. Tell what change was made in each of the above examples before dividing.

When the divisor contains a decimal, make the divisor a whole number by multiplying both divisor and dividend by 10, 100, 1000, etc.

In dividing a decimal by a whole number, the decimal point in the quotient stands in the same column as the decimal point in the dividend.

The first step in division of decimals is to write the decimal point in the quotient.

Solve at sight:

8. 4.8 ÷ 6
9. .24 ÷ 4
10. .045 ÷ 5
11. 56 ÷ .7
12. 2.7 ÷ .3
13. .54 ÷ .9

Divide:

14. .085 by 17
15. .0096 by 8
16. .0027 by 18
17. 7.02 by 6.5
18. .308 by .35
19. 551.2 by 2.08
20. 44.64 by 2.88
21. 633.6 by 6.4
22. .06 by .001
23. 15.129 by 12.3
24. 204 by .085
25. 262.5 by 5.25
26. 22.5 by 1800
27. 130.8 by .01
28. 30.03 by .003
29. 671 by .305
30. 70.62 by 160.5
31. 1943.5 by .25
32. .08181 by 9.09
33. .1728 by 48
34. 1728 by .96
35. 525.6 by .36
36. 63.36 by .32
37. 156.6 by .58
38. .4662 by 74
39. 2.7202 by 58
40. .2546 by .67
41. .8892 by 247
42. .138 by 276
43. 346.56 by 6.08

44. 408 by .017
45. 186.02 by 100
46. 3.92 by .056
47. 15.04 by .08
48. 9.09 by .003
49. 43.008 by 4.096
50. 14.124 by 32.1
51. 56.088 by 123
52. 13.31 by .011
53. .0576 by .36
54. .0084 by .035
55. .03648 by .006
56. .0616 by .007
57. .0432 by 36

WRITTEN PROBLEMS

1. A farmer sold five loads of hay weighing as follows: .45 ton, .625 ton, .54 ton, .375 ton, and $\frac{4}{5}$ ton. How many tons did he sell?

2. The sides of an irregular field measure 30.65 rods, 42.15 rods, 18.07 rods, 19.05 rods, 20.02 rods, 17.024 rods. How many rods less than a mile (320 rods) is the distance around the field?

3. From a hogshead of molasses containing 84 gallons there are drawn 6.5 gallons, 12.75 gallons, 15.625 gallons, 20 gallons, .75 gallon, and 3.25 gallons. How many gallons are left?

4. Mr. Brown sold land as follows: .78 acre, $\frac{1}{2}$ acre, $6\frac{3}{8}$ acres, $.87\frac{1}{2}$ acre, and $.7\frac{1}{2}$ acre. How many acres did he sell?

5. The statute knot used in measuring distances on the sea is 1.151 miles. What is the speed in miles of a steamship making 20.5 knots an hour?

6. How long a bolt will be required to fasten together two planks, one 3.5 inches thick and the other 2.25 inches thick, allowing .375 of an inch for the head of the bolt and .375 of an inch for the nut?

7. A surveyor finds the length of a schoolhouse lot to be 69.85 yards. Express the length in rods. (5.5 yards make 1 rod.)

8. How many yards in 82.3 rods?

9. The French franc is worth 19.3 cents. Five francs are how much less than a United States dollar?

10. How many French franc pieces are equal to $3.86 in our money?

11. At the rate of .9 of a dollar per thousand cubic feet, Mr. Wilson's gas bill for November was $3.15. How many thousand cubic feet were used?

12. A bushel of wheat occupies 1.25 cubic feet of space. How many bushels will a bin hold whose capacity is 52.5 cubic feet?

13. The English pound sterling is worth $5.8665 in United States money. Five English pounds sterling are worth how much more than 5 5-dollar United States gold pieces?

14. A quart of water weighs 2.08 pounds (nearly). What is the weight of the water in a kettle holding 2.5 gallons?

15. Milk is 1.03 times as heavy as water. What is the weight of a gallon of milk?

16. A clerk in a store receives $2.50 a day. The head of the department receives .2 more, and the errand boy .4 as much as the clerk. How much does the head of the department receive? How much does the boy receive?

17. How many yards of cloth in 36 bolts of 40.75 yards each?

DRILL EXERCISE

	A	*B*	*C*	*D*	*E*
1.	$\frac{1}{8}$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{16}$
2.	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{4}{5}$	$\frac{7}{8}$	$\frac{3}{5}$
3.	$\frac{1}{5}$	$\frac{7}{10}$	$\frac{3}{50}$	$\frac{20}{25}$	$\frac{7}{100}$
4.	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{10}$	$\frac{1}{16}$	$\frac{3}{20}$
5.	$\frac{1}{10}$	$\frac{2}{5}$	$\frac{7}{8}$	$\frac{49}{50}$	$\frac{1}{40}$

1. At sight, subtract each fraction from 10.

2. Find the sum of each column; of all the columns.

3. Find the sum of each row; of all the rows.

4. Make a table similar to that above, expressing each fraction as a decimal.

5. Find the sum of each column; of all the columns.

6. Find the sum of each row; of all the rows.

COUNTING *Oral*

12 units = 1 dozen (doz.)
12 dozen = 1 gross (gr.)
20 units = 1 score
24 sheets of paper = 1 quire
20 quires = 1 ream

500 sheets are frequently called a ream.

Paper is quite generally sold by the pound.

1. How many dozen in 75 eggs? In 100 eggs?

2. How many oranges in $3\frac{3}{4}$ dozen? In $2\frac{5}{6}$ dozen?

3. Compare a dozen with a score. A score with a dozen.

4. How old is a man who is "threescore years and ten"?

5. How many score in a century?

6. At 2 cents each, how much will a gross of pencils cost?

7. How many gross of pencils in 8 boxes of 6 dozen each?

8. How many sheets in a ream?

9. How many envelopes must be bought to match $2\frac{1}{2}$ quires of letter paper?

10. What is the cost of $\frac{1}{4}$ of a ream of paper at 20 cents a quire?

11. At $2 a ream, what is the cost of 6 quires of paper?

LINEAR MEASURE

1. Repeat the table of linear measure.

2. 1 mile = —— rods = —— yards = —— feet = —— inches.

3. What are the dimensions in feet of a house lot 2 rods by 4 rods? What is the perimeter of the lot in feet?

4. Express as parts of a mile the dimensions of a lot 80 rods by 40 rods. Express its perimeter in miles.

5. A fathom is 6 feet. A cable length is 120 fathoms. A ship is anchored 3 cable lengths from shore in $15\frac{1}{2}$ fathoms of water. Express distances in feet.

6. Noah's ark was 300 cubits long, 50 cubits wide, and 30 cubits high. A sacred cubit is 1.824 feet. Give the dimensions in feet.

7. A military pace is 2½ feet. At 120 paces a minute, how long will it take a regiment to march a mile?

8. How many feet of wire must be used in building a framework for grapevines, if there are 4 rows, each 3 rods long and 6 wires high?

9. How many rails 30 feet long will be required to lay a railroad track ½ of a mile long?

10. An aviator reaches a height of 11,474 feet. Express the height in miles.

11. A stone contractor agreed to furnish 280.5 feet of curbing at $14.20 per rod. What was the total price?

12. Five house lots side by side measure, respectively, 70 ft. 6 in., 62.75 ft., 4 rd., 22¾ yd., and 57½ ft. What is the total frontage in feet?

13. A map is drawn on a scale of 1″ to 37½ miles. What is the distance between two places 3⅞ inches apart?

14. On another map, ¼″ represents 1 rod. How many feet does a line 1′ 3″ long represent?

15. Letting 1″ represent 160 rods, draw a line to represent the distance between two places 1¼ miles apart.

16. A contractor who was to build a piece of railroad track $2\frac{3}{16}$ miles long was obliged to stop work after completing 525 rods. What part of the work was unfinished?

17. At $4.92 per bale of 20 rods, how much must Mr. Jenkins pay for the wire fencing necessary to inclose his pasture, 14 rods long and 6 rods wide?

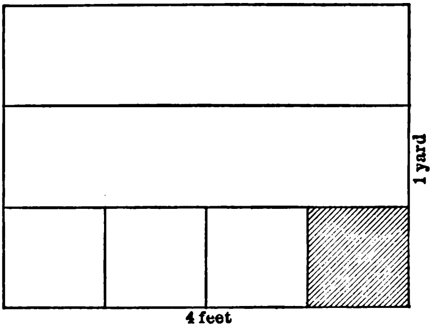
SQUARE OR SURFACE MEASURE

Write the table of square measure.

1 acre = —— square rods = —— square yards = —— square feet = —— square inches.

The area of a figure is the number of square units it contains.

The figure represents a surface 4 feet long and 1 yard wide. What is its area? A surface 4 feet by 1 yard is a surface 4 feet by 3 feet.



The shaded part represents the unit of measurement, — 1 square foot.

If the figure were 4 feet long and 1 foot wide, it would contain how many square feet? Since it is 3 feet wide, it contains 3 times 4 square feet, or 12 square feet.

Steps taken:

First. Determine the unit of measurement.

Second. Find the number of these units in one row.

Third. Multiply the number of units in one row by the number of rows.

Short method: $3 \times 4 = 12$.

In finding areas the dimensions must be expressed in the *same* unit of measurement (inches, feet, yards, etc.).

PROBLEMS *Oral*

1. What measurements must be made to find the area of one side of a sheet of examination paper? How is the area found?

2. Remembering that the multiplier is an abstract number simply showing how many times, explain just how to find the area of a piece of paper 8 in. long and 6 in. wide.

3. A geography covers 96 square inches on the table. It is 8 inches wide. How long is it?

4. A field is 9 rods long and $\frac{2}{3}$ as wide. What is its area? What is its perimeter?

5. One board is 16 in. by 4 in. Another is 8 in. square. Compare their areas. Compare their perimeters.

6. The area of a square lot is 49 square rods. What is its perimeter?

7. The perimeter of a square lot is 48 rods. What is its area?

8. A field is 8 rods long. It contains 32 square rods. How many rods of fence will inclose it?

9. What is meant by 2 square inches?

10. What is meant by a 2-inch square?

11. Mr. Chase has two square fields. One contains 64 square rods. The distance around the other is 64 rods. Compare their areas. Compare their perimeters.

PROBLEMS *Written*

1. What are the dimensions in feet of a 12-inch square? What is its perimeter in feet? Its area in square feet?

2. Make and fill out a table like the following:

SQUARES	DIMENSIONS IN FEET	PERIMETER IN FEET	AREA IN SQUARE FEET
12-inch	1 foot	4 feet	1 square foot
24-inch			
36-inch			

Continue with 48-inch, 60-inch, 72-inch, 84-inch, 96-inch, 108-inch, 120-inch, 132-inch, 144-inch squares.

3. Mr. James has a lot of land containing 16 square rods. Express its area in acres.

4. Mr. Brown has a lot that is 16 rods square. Express its area in acres.

5. A lot 108 ft. by 75 ft. is sold at 12½ cents a square foot. How much is received for it?

6. A lot of land is 66 feet wide and 1½ times as long. (1) How many rods of fence will be required to inclose it? (2) What is the value of the lot at 18 cents a square foot?

7. How many feet of weather molding are necessary for a door 6′ 10″ by 3′ 4″? What is the area of the opening when the door is wide open?

Find the number of acres in lots of these dimensions:

	LENGTH	WIDTH		LENGTH	WIDTH
8.	32 rd.	24 rd.	**9.**	60 rd.	60 rd.
10.	20½ rd.	18 rd.	**11.**	12½ rd.	10½ rd.
12.	1⅞ rd.	1⅖ rd.	**13.**	3⅕ rd.	3⅕ rd.
14.	4.8 rd.	2.25 rd.	**15.**	½ rd.	½ rd.
16.	369 ft.	110 ft.	**17.**	440 ft.	396 ft.
18.	24 rd.	24 rd.	**19.**	25.5 rd.	25.5 rd.

20. Find the perimeters of the lots in examples 8 to 19.

21. A lot containing 2 A. 112 sq. rd. was cut up into 18 equal house lots. How many square rods in each lot?

22. Mr. Jackson paid $8000 for a lot 40 rd. by 30 rd. At the same rate what ought he to pay for another lot one half as long and one half as wide?

23. The garden of the Harvard school is 80 ft. by 60 ft. Each pupil has a plot 10 ft. by 5 ft. Allowing one sixth of the whole area for paths, how many individual gardens are there?

Find the areas of the following rectangles:

	LENGTH	WIDTH		LENGTH	WIDTH
24.	18′	12′	**25.**	15 in.	12½ in.
26.	20′	15′	**27.**	24 ft.	18 ft. 6 in.
28.	16′ 6″	14′ 6″	**29.**	7½ ft.	7½ ft.
30.	1¾″	1¾″	**31.**	¼ rd.	4⅛ yd.
32.	1′ 6″	⅔′	**33.**	4¼ ft.	20 in.
34.	3′ 3″	2′ 6″	**35.**	48 ft.	15 yd.
36.	1¾′	1⅔′	**37.**	60 yd.	40 yd. 2 ft.
38.	1′ 8″	1′ 3″	**39.**	½ yd.	½ ft.
40.	15′ 6″	14′	**41.**	15¾ ft.	15¾ ft.
42.	12′ 8″	10′ 6″	**43.**	1.2 rd.	7.75 yd.

44. Express the perimeters of the above rectangles in inches, feet, or yards, as directed.

45. If you were given your choice of three lots — 36 rd. by 9 rd., 27 rd. by 12 rd., and 18 rd. by 18 rd. — on condition that you pay for fencing, which would you choose?

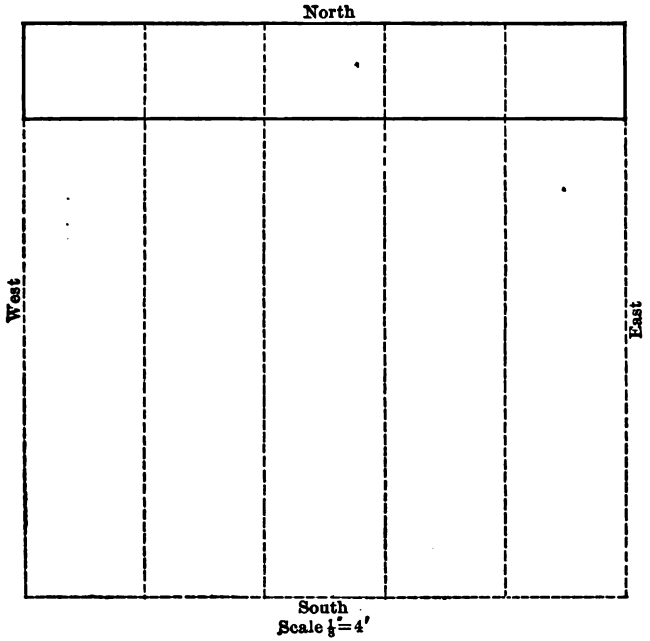
46. The cost of fencing a lot at 22 cents per foot was $73.92. What was the perimeter of the lot?

47. A chess board is 8 squares long and 8 squares wide. Each square is 1½ inches long. What is the area of the board? The distance around its edge?

48. What is the area of a table 2 ft. 8 in. by 2 ft. 8 in.?

A POULTRY HOUSE AND YARDS

The diagram represents the ground plan of a poultry house and yards.



Find:

1. The area covered by the house; by each compartment; the number of square feet of floor space to a fowl if there are 50 in a compartment.

2. The area of all the yards; of each yard; the yard space allowed to each fowl.

3. The total area covered by house and yards; the part of the total area covered by the house; by the yards.

4. The number of feet of fencing required; the number of rods.

5. The number of 5-rod bales of fencing that must be bought and the cost at $1.60 per bale. (The dealer will not sell a part of a bale.)

6. The number of posts required to support the netting about the yards if placed 10 feet apart on the south side and 8 feet apart on the lines running north and south, if the netting is fastened directly to the house.

7. The cost of the posts at 13 cents each; total cost of netting and posts.

USE OF SIGNS

1. What operations must be performed first in solving
$36 \div 2 + 3 \times 2$? In $20 - 4 \times 2 + 6 \div 3$?
$24 \div 2 + 4 = 12 + 4 = 16$. $24 \div (2 + 4) = 24 \div 6 = 4$.

When several terms are inclosed in a parenthesis, solve the part in parenthesis first.

Solve:

2. $13 + 3 \times 2$. 3. $(13 + 3) \times 2$
4. $16 - 2 \times 4$. 5. $(16 - 2) \times 4$.
6. $12 + 8 \div 4$. 7. $(12 + 8) \div 4$.
8. $35 \div 5 + 2$. 9. $35 \div (5 + 2)$.
10. $48 \div 2 + 4 - 3$. 11. $48 \div (2 + 4 - 3)$.

Brackets, [ ], braces, { }, and the vinculum, $\overline{\quad\quad}$, have the same force as the parenthesis.

12. $24 \div 6 - 2$. 13. $24 \div [6 - 2]$.
14. $24 \times 2 + 4$. 15. $24 \times \{2 + 4\}$.
16. $48 \div 12 + 4$. 17. $48 \div \overline{12 + 4}$.
18. $36 \div \overline{3 + 6} - 3 \times 2$. 19. $60 \div (10 + 5) + 8 \times 3$.

20. Show that $83 - 5 \times 4$ is equal to $(10 + 11) \times 3$.

21. Which is greater, $4 + 8 \times 5$ or $\overline{4 + 8} \times 5$?

22. What is the sum of $54 \div 6 + 12$ and $54 \div (6 + 12)$?

23. How many times is
$42 \div \overline{2 + 4}$ contained in $(32 - 4) \times 2$?

24. If $54 \div (6 \times 9)$ is taken from $54 \div 6 \times 9$, what is left?

25. What is the product of $20 \times 3 + 3$ and $20 \times \overline{3 + 3}$?

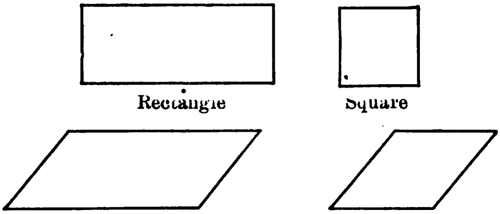
26. Divide $\overline{40 - 8} \times 2$ by $15 \times 2 + 2$.

27. Find the sum, difference, product, and quotient of $56 + 98 \div 14 - 2 \times 1\frac{1}{2}$ and $\overline{56 + 98} \div 14 - 2 \times 1\frac{1}{2}$.

QUADRILATERALS

1. What is a quadrilateral?

PARALLELOGRAMS



Rectangle Square

Rhomboid Rhombus

2. What name is given to quadrilaterals having two pairs of parallel sides?

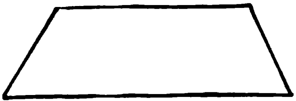
3. Name the different kinds of parallelograms.

4. Describe a rectangle. A square. A rhomboid. A rhombus.

NOTE. Any correct description should be accepted. The main thing is to be sure that pupils distinguish clearly the different figures.

5. What is the altitude of a parallelogram?

6. Tell how to find the area of a rectangle. Of any parallelogram.


TRAPEZOID

7. What name is given to quadrilaterals having one pair of parallel sides?

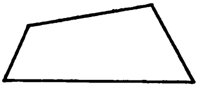
8. What is the altitude of a trapezoid?

9. Draw a trapezoid on paper. Cut it out and fold so that the parallel sides coincide. Cut along the crease; arrange the parts to form a parallelogram. What is the base of the parallelogram? Its area? What is the area of the trapezoid?

The area of a trapezoid is equal to one half the product of its altitude and the sum of its parallel sides.

Find the areas of trapezoids of these dimensions:

	PARALLEL SIDES	ALTITUDE
10.	20 in. and 12 in. $\frac{(20 + 12) \times 8}{2} =$	8 in.
11.	16 in. and 18 in.	9 in.
12.	18 in. and 15 in.	10 in.
13.	32 ft. and 10 ft.	12 ft.
14.	1 ft. 8 in. and 1 ft.	16 in.
15.	2 ft. 8 in. and 1 ft. 4 in.	1 ft. 6 in.
16.	8.8 ft. and 16 ft.	1.5 ft.
17.	6.75 ft. and 4.25 ft.	3 ft.
18.	$12\frac{3}{4}$ ft. and $8\frac{2}{3}$ ft.	6 ft.
19.	$18\frac{1}{3}$ ft. and $16\frac{2}{3}$ ft.	$8\frac{1}{2}$ ft.



TRAPEZIUM

20. What name is given to quadrilaterals having no two sides parallel?

21. Draw a trapezium; draw one of its diagonals. Cut out the trapezium; separate it into two triangles by cutting along the diagonal. Find the area of each triangle. Of both triangles. What is the area of the trapezium?

22. Tell how to find the area of a trapezium.

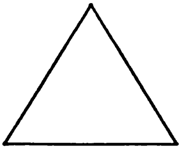
23. Draw other trapeziums and find their areas.

TRIANGLES

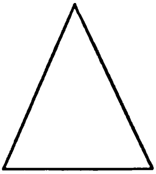
1. What is a triangle?

Triangles compared:

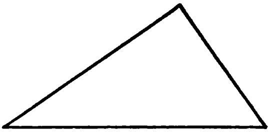
I. With respect to their sides.



EQUILATERAL



ISOSCELES



SCALENE

2. Describe an equilateral triangle.

3. What is the perimeter of an equilateral triangle, whose sides are each 7 inches? $2\frac{1}{2}$ inches? $3\frac{3}{4}$ inches? $12\frac{1}{2}$ rods?

4. The perimeter of an equilateral triangular flower bed is $16\frac{1}{2}$ feet. What is the length of one side?

5. Describe an isosceles triangle.

6. What is the perimeter of an isosceles triangle whose equal sides are each 6 feet and whose base is 4 feet?

7. The perimeter of an isosceles triangle is 20 inches; its base is 6 inches. What is the length of each of the other sides?

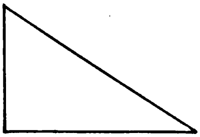
8. The base of an isosceles triangle is $2\frac{1}{2}$ rods; one of the equal sides is $5\frac{1}{4}$ rods. What is the perimeter?

9. Describe a scalene triangle.

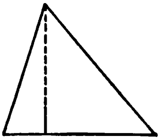
10. The sides of a scalene triangle are $8\frac{1}{2}$ inches, $5\frac{3}{4}$ inches, and $6\frac{1}{2}$ inches, respectively. Find the perimeter.

11. The perimeter of a scalene triangle is $20\frac{1}{2}$ yards. Two sides are, respectively, 30 feet and 12 feet. Find the third side.

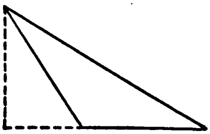
II. With respect to their angles.



RIGHT-ANGLED



ACUTE-ANGLED



OBTUSE-ANGLED

12. Describe a right-angled triangle. An acute-angled triangle. An obtuse-angled triangle.

13. What is the altitude of a triangle?

FINDING AREAS OF TRIANGLES

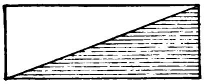


FIGURE 1

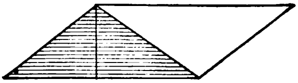


FIGURE 2

1. Tell how to find the area of figure 1. Of the shaded part. Of the shaded part of figure 2.

The area of a triangle is equal to one half the product of its base and its altitude.

2. What is the area of a triangular piece of cardboard whose base is 6½ inches and whose altitude is 3½ inches?

3. A patchwork bedspread is made up of 378 pieces, each in the shape of a right-triangle, whose base is 4 inches and whose altitude is 4 inches. How many square feet does it cover? How many square yards?

4. The two short sides of a corner cupboard are each 1 foot 8 inches, inside measurement. What is the area of the bottom shelf?

5. Mr. Jamieson has a wood lot in the shape of a triangle. One side is 40 rods, and the shortest distance from this side to the opposite angle is 36 rods. How many acres in the lot?

6. A farmer has a triangular pasture whose sides are each 80 rods. How much will it cost to fence it with wire fencing at $2.88 per bale of 10 rods?

7. The distance from the middle of one of the sides to the opposite angle of the pasture in problem 6 is 89.44 rods. What is the area in square rods? In acres?

ORAL EXERCISES

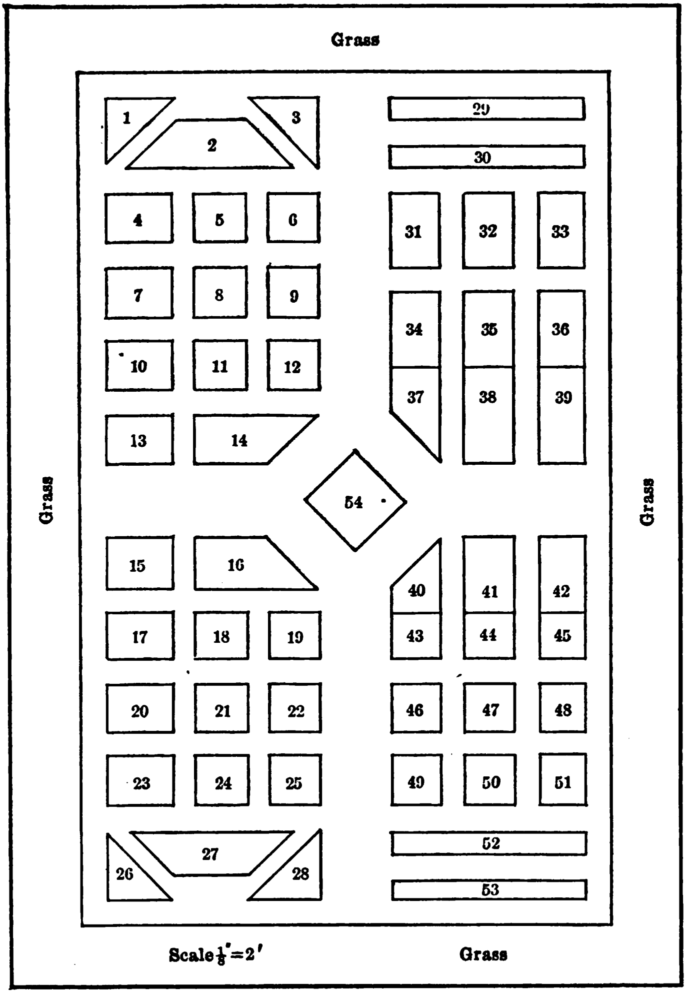
1. Find the cost per dozen when the price of one is:
6¼¢ $¾ 37½¢ $1.25 $1⅔ $2⅕

2. Find the price of one when the cost per dozen is:
60¢ 40¢ $½ $1.08 $20 $37.50

3. Find the rate per dozen when 4 articles are sold for 15¢; 6 for 75¢; 3 for 10¢; 2 for $1.25; 8 for 20¢.

4. Tell the price of one when the cost per hundred is:
$3 $2.50 $1¼ $8.50 $40 $50

THE JEFFERSON SCHOOL GARDEN



The garden is planted as follows:

FLOWERS

Asters (pink), 12, 19.
Asters (purple), 4, 23.
Asters (white), 6, 25, 10, 17.
Daisies, 7, 20.
Foliage plants, 54.
Forget-me-nots, 9, 22.
Geranium (red), 2, 27.
Nasturtiums, 13, 15.
Pansies, 14, 16.
Poppies, 8, 21.
Shrubs, 1, 3, 26, 28.
Sweet Peas, 5, 24.
Violets, 11, 18.

VEGETABLES

Beans, 36, 51.
Beets, 39, 45.
Cabbages, 35, 50.
Carrots, 37, 40.
Celery, 30.
Cucumbers, 42.
Lettuce, 34, 46.
Onions, 32, 44.
Parsley, 49.
Parsnips, 48.
Potatoes, 38, 41.
Squashes, 52.
Sweet corn, 29, 53.
Tomatoes, 33, 47.
Turnips, 31, 43.

NOTE. The following exercise may be assigned for home work.

Copy the plan of the garden on a scale of $\frac{1}{4}''$ to $3'$. Write in each plot its area as found.

1. The garden is in the form of what figure? What are its dimensions? Its area?

2. At $1\frac{1}{2}$¢ a foot, how much will it cost to inclose the garden with a wire fence?

3. Find the area covered by the grass plot.

THE FLOWER GARDEN

4. What is the shape of the bed of foliage plants? Its area? Give the numbers of other plots of the same shape.

5. What name is given to the plots devoted to shrubs? Find the area of each plot. Of all.

6. What is the shape of the beds of red geraniums? The area of each? Of both?

7. Of what shape are the beds of pansies? What is the area of each? Of both?

8. Of what shape are the beds of purple asters? What is the area of each? Give the numbers of all other beds of the same shape and area.

9. Find the area devoted to pink asters; to white asters; to daisies; to forget-me-nots; to nasturtiums; to poppies; to sweet peas; to violets.

10. Find the total area of the flower beds.

THE VEGETABLE GARDEN

11. Find the difference in area of the two plots of potatoes. The area of both plots.

12. What is the total area devoted to onions?

13. Of what shape are the plots of carrots? What is the area of each plot? Of both?

14. Of what shape are the patches of sweet corn? The area of each?

15. Find the area devoted to beans; to beets; to cabbages; to celery; to cucumbers; to lettuce; to parsley; to parsnips; to squashes; to tomatoes; to turnips.

16. At ½ ¢ a foot, how much will it cost to place wire netting around the cucumber and the squash beds?

17. Find the total area of the vegetable beds.

THE WHOLE GARDEN

18. Find the total area of flower and vegetable beds.

19. Find the total area of garden and grass plots.

20. From the answers to problems 1 and 19, find the area covered by paths.

PLASTERING

Plastering is usually estimated by the square yard.

In estimating work to be done, every contractor follows a rule which experience has taught him to be reliable. The general custom is to deduct from the total area of the walls and ceiling one half the area of openings, reckoning to the nearest yard.

In solving problems, make your practice in regard to methods and prices conform to local conditions. Builders are always willing to furnish any information desired.

1. How many square yards in the walls and ceiling of a room 21 ft. long, 18 ft. wide, and 12 ft. high?

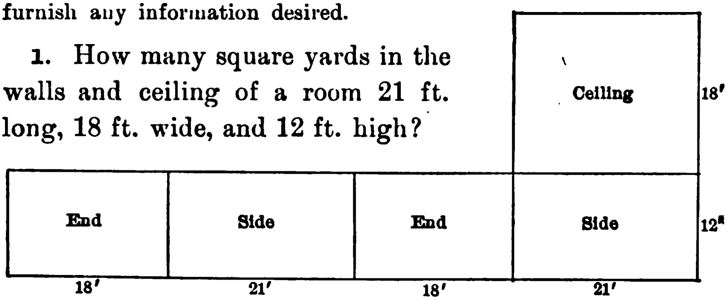


DIAGRAM OF WALLS AND CEILING

This may be illustrated by a pasteboard box without cover.

The walls form a rectangle whose length is the perimeter of the room. What are the dimensions of the rectangle formed by the walls? By the ceiling?

78 × 12 = 936, no. of sq. ft. in walls
21 × 18 = 398, no. of sq. ft. in ceiling
1334, no. of sq. ft. in walls and ceiling
9)1334
$148\frac{2}{9}$, no. of sq. yd. in walls and ceiling

The plasterer would call this 148 sq. yd.

Find the number of square yards in the walls and ceiling of the following rooms:

	LENGTH	WIDTH	HEIGHT		LENGTH	WIDTH	HEIGHT
2.	16 ft.	12 ft.	8 ft.	**3.**	17′	15′	9′
4.	20 ft.	18 ft.	12 ft.	**5.**	18′	16′	$9\frac{1}{2}$′
6.	15 ft.	15 ft.	9 ft.	**7.**	18′ 6″	16′	10′
8.	30 ft.	28 ft.	12 ft.	**9.**	15′ 6″	13′ 6″	9′

10. At 3½ cents a foot, what will be the cost of picture molding for each of the above rooms?

11. At 25 cents a square foot, what will it cost to lay a hardwood floor in each of the above rooms?

12. A schoolroom is 30′ × 24′ × 12′. Find the cost of putting a blackboard 4 feet wide on the long side at 22½ cents per square foot.

13. At 30¢ a square yard, find the cost of plastering the walls and ceiling of a room 18 ft. long, 15 ft. wide, and 9 ft. high, making no allowance for openings.

14. Find the cost of plastering the room in problem 13, allowing 96 sq. ft. for openings.

Find the cost of plastering the walls and ceilings of the following rooms:

	LENGTH	WIDTH	HEIGHT	ALLOWANCE FOR OPENINGS	COST PER SQ. YD.
15.	15′	12′	9′	⅛ total area	28¢
16.	21′	18′	10′	⅕ total area	25¢
17.	24′	18′	10′	20 sq. yd.	30¢
18.	27′	21′	12′	24 sq. yd.	24¢
19.	12′	10′	8½′	none	28¢

20. At 40 cents per square yard, what is the cost of plastering a room 15′ × 12′ × 8′, allowing for 2 openings 6′ × 3′, and 2 openings 8′ × 3½′? Deduct one half the area of openings.

21. A room is 18′ 9″ long, 13′ 6″ wide, and 10′ high. There are 2 openings 7′ × 3½′, and 3 openings 6′ × 3′. Find the area the plasterer reckons.

22. Make out the bill to the city for plastering the schoolroom at the local rates.

CARPETING

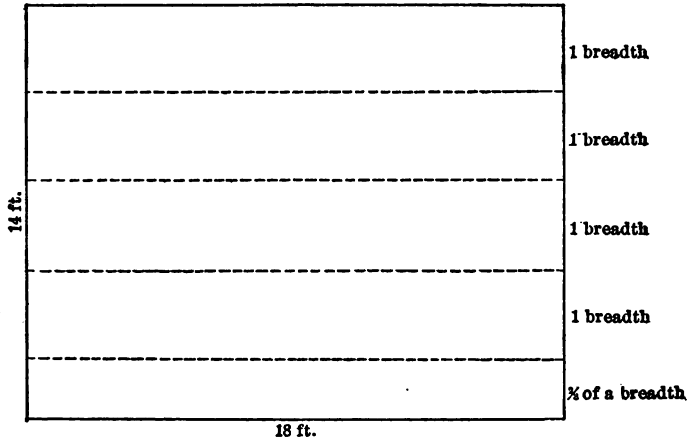
Carpeting is of varying widths, but is sold by the linear yard. In buying, the width must always be kept in mind. The more common widths are: ingrains 1 yard, and others ¾ of a yard.

The breadths or strips are usually laid lengthwise of the room. In the following problems follow this custom unless otherwise directed.

Since the dealer never cuts or splits a breadth, sometimes more carpeting must be purchased than is absolutely necessary to cover the floor. There is also frequently some waste in matching for which the purchaser has to pay.

Oilcloth, linoleum, etc., are sold by the square foot or the square yard.

1. How much carpeting is required for a room 18 ft. long and 14 ft. wide?



Since the carpet is laid lengthwise of the room, as many breadths or strips must be bought as the width of the carpet (3 feet) is contained times in the width of the room, 14 feet. 14 ÷ 3 = 4⅔. Since the dealer never sells part of a breadth, 5 breadths must be purchased. Each strip runs the length of the room, 18 feet. 5 strips = 90 feet. 90 feet = 30 yards.

2. How many yards of carpeting must be bought for a room 16 ft. long and 13 ft. wide?

3. If an allowance of one foot on each strip except the first is made for matching, how many yards of carpeting will be required for a room 24 ft. long and 18 ft. wide?

4. About a room 18½ ft. by 16½ ft. is a hardwood border 1 ft. 6 in. wide. What are the dimensions of a rug just large enough to cover the rest of the floor?

5. I wish to carpet a room 21 ft. by 17 ft. in the most economical way. Which way ought the strips to run?

6. Make out the bill for carpeting a room 16 ft. by 15 ft. with Brussels carpet at $1.10 per yard.

7. At $1.50 per yard, what will it cost to carpet a room 19 ft. by 16 ft., if a total of 1½ yards is wasted in matching?

8. In a certain room the carpet is to be laid crosswise. The room is 5⅔ yd. long and 4¾ yd. wide. How much carpeting will be required?

9. How many yards of straw matting 1 yd. wide must be bought for a room 17 ft. 6 in. long and 11 ft. 10 in. wide?

10. At $1.35 a square yard, what is the cost of a linoleum for a kitchen 13½ ft. by 8 ft.?

Find the cost of carpeting the following rooms:

	DIMENSIONS OF ROOMS		WIDTH OF CARPET	PRICE PER YARD
	Length	Width		
11.	17 ft.	14 ft.	1 yd.	$0.95
12.	18 ft.	15 ft.	¾ yd.	$1.45
13.	16⅔ ft.	14½ ft.	1 yd.	$1.10
14.	19 ft.	17¾ ft.	¾ yd.	$1.62½

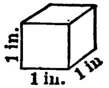
CUBIC OR VOLUME MEASURE

1. Write the table of cubic measure.

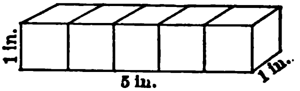
A load of earth, sand, etc., is a cubic yard.

The volume of a solid is the number of cubic units it contains.

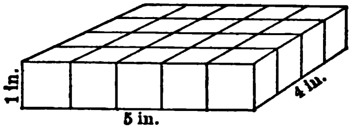
2. What is the volume of a solid 5 in. long, 4 in. wide, and 3 in. high?



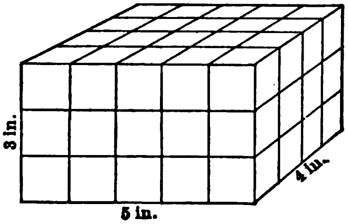
Unit of measurement



1 row of cubic units



1 layer of cubic units



The unit of measurement is 1 cubic inch. How many of these cubic units in 1 row? In 1 layer? In all the layers?

Steps taken:

First. Determine the unit of measurement.

Second. Find the number of cubic units in 1 row.

Third. Multiply the number of cubic units in 1 row by the number of rows.

Fourth. Multiply the number of cubic units in 1 layer by the number of layers.

In finding volumes the three dimensions must be expressed in the *same* unit of measurement (inches, feet, yards, etc.).

1. How many dimensions has a cube? How do they compare?

2. How many faces has a cube? What is their shape? How do they compare in size?

3. What is the length in feet of a 12-inch cube? The area of one of its faces in square feet? Of all its faces? Its volume in cubic feet?

4. Make and fill out a table like the following:

CUBES	LENGTH IN FEET	AREA OF FACES IN SQUARE FEET	VOLUME IN CUBIC FEET
12-inch	1 foot	6 square feet	1 cubic foot
24-inch			
36-inch			

Continue with 48-inch, 60-inch, 72-inch, 84-inch, 96-inch, 108-inch, 120-inch, 132-inch, 144-inch cubes.

Find the volumes of these rectangular prisms:

	LENGTH	WIDTH	HEIGHT		LENGTH	WIDTH	HEIGHT
5.	6 ft.	5 ft.	7 ft.	**6.**	15 in.	12 in.	8 in.
7.	2 ft.	15 in.	9 in.	**8.**	4 ft.	1 yd.	1½ ft.
9.	1½ yd.	1½ ft.	18 in.	**10.**	2 yd.	2¼ ft.	18 in.
11.	27 in.	1½ ft.	15 in.	**12.**	1¼ yd.	1½ ft.	2½ ft.
13.	2.5 ft.	2.5 ft.	2.5 ft.	**14.**	3.5 ft.	2.25 ft.	2 ft.

15. What is the volume of a toy building block 2 in. by 2 in. by 2 in.? What is the capacity of a box that will exactly hold 12 of them? How many can be placed in a box 6 in. by 6 in. by 6 in.?

16. Which is larger, a 4-foot cube or 4 cubic feet? How much?

17. What is the volume of a crate that will hold 24 boxes of butter each 1 ft. long, 4 in. wide, and 4 in. high?

18. How many cubic feet in a stone step 4 ft. long, 15 in. wide, and 8 in. high?

19. A camphor chest 4 ft. long and 3 ft. wide stands 2 ft. 6 in. high. Find its volume.

20. A block of ice cream 1 ft. long, 4 in. wide, and 3 in. thick is served to 12 persons. What are the dimensions of each slice?

21. A man buys 14 bales of hay, each 4 ft. long, 2½ ft. wide, and 2 ft. thick. How much space will they occupy in his barn? What is their value at $24 a ton, if their average weight is 150 pounds?

22. A stone pillar is 2½ ft. square and 8½ ft. high. What is its volume?

23. What is the capacity of a tank whose width is 3 ft., whose height is ½ of its width, and whose length is twice its width?

24. Draw a diagram of the faces of a 4-inch cube, on a scale of ½″ to 1″.

25. Draw a diagram of the faces of a rectangular prism 4 ft. × 3 ft. × 2 ft. Scale 1″ = 1′.

26. Find the surface area of a block of wood 3 inches square and 4 inches high.

27. How many square feet of copper will be required to line the bottom and sides of a tank 4 ft. long, 3 ft. wide, and 2 ft. deep?

28. At 6 cents a square foot, how much will it cost to line with zinc a tank 3 ft. long, 2½ ft. wide, and 2 ft. deep?

29. At 75 cents a cubic yard, what is the cost of digging a cellar 24 ft. by 16 ft. by 7 ft.?

30. How many loads of earth must be carted to fill a ditch 48 ft. long, 4 ft. deep, and of an average width of 9 ft.?

WOOD MEASURE

1. A pile of wood 1 ft. by 4 ft. by 4 ft. is a cord foot. How many cubic feet in a cord foot?

2. A pile of wood 8 ft. by 4 ft. by 4 ft. is a cord. How many cubic feet in a cord? How many cord feet in a cord?

3. Write the table of wood measure.

Wood, bark, etc., are generally sold by the cord.

4. How many cords of wood in a pile 20 ft. long, 4 ft. wide, and 6 ft. high?

5. A pile of wood contains 1728 cubic feet. How many cords?

6. A farmer has a wagon 12 ft. long and 4 ft. wide. How high must he pile the wood in loading a cord?

7. At $7.50 a cord, what is the value of a pile of 4-foot wood, 18 ft. long and 8 ft. high?

Find the number of cords of 4-foot wood in the following lots:

	LENGTH	HEIGHT		LENGTH	HEIGHT
8.	15 ft.	6 ft.	**9.**	18 ft.	4 ft.
10.	22 ft.	5 ft.	**11.**	16 ft.	3½ ft.
12.	24 ft.	6½ ft.	**13.**	20 ft.	4½ ft.
14.	12 ft. 6 in.	8 ft.	**15.**	16 ft.	6½ ft.

16. A railroad car 30 ft. long and 8 ft. wide is loaded 6 ft. high with wood. What is the value of the wood at $8.75 per cord?

MASONRY

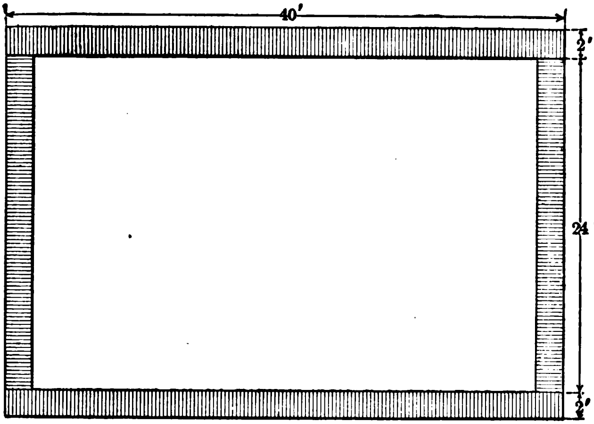
Masonry and bricklaying are usually measured by the cubic foot.

In estimating the number of cubic feet in the walls of buildings, the outside dimensions are taken. This counts the corners twice, but the extra labor required to build the corners offsets their double counting.

In estimating the material required, the corners are counted once.

Common bricks are 8″ × 4″ × 2″, and, allowing for space occupied by mortar, 22 of them will lay a cubic foot.

1. The outside dimensions of a foundation wall are 40 ft. by 28 ft. The cellar is 9 ft. deep and the walls are 2 ft. thick. Find the number of cubic feet of masonry; the number of cubic feet of material required.



HINT. Reckoning the outside dimensions, the cellar walls are equivalent to a single wall 40′ + 28′ + 40′ + 28′, or 136′, in length.

Reckoning exactly, the cellar walls are equivalent to a single wall 40′ + 24′ + 40′ + 24′, or 128′, in length.

2. How many cubic feet of masonry in a wall 60 ft. long, 5 ft. high, and 1½ ft. thick? How many bricks are required to build this wall?

3. How many cubic feet of stone will be required to build a foundation for a storehouse $100' \times 60'$, if the walls are $4'$ thick and $10'$ high?

4. At $0.90 per cubic foot, what will be the expense of building a brick wall $2\frac{1}{2}$ ft. high and $1\frac{1}{2}$ ft. thick on the two street sides of a corner lot 120 ft. by 80 ft.?

5. At $2.75 per cubic yard, what will it cost to build the foundation of a house 36 ft. by 24 ft., outside measurements, if the cellar is to be 9 ft. high and the wall $1\frac{1}{2}$ ft. thick?

6. When bricks are $$8\frac{1}{2}$ per M, what will be the cost of the bricks necessary to build a partition wall 48 ft. long, 27 ft. wide, and 8 in. thick?

7. The north side of a greenhouse is a brick wall 24 ft. long, 12 ft. high, and 8 in. thick. Find the surface area of the side within the house; the number of cubic yards of masonry in the wall.

BOARD MEASURE

Wood used in building is called *lumber*, and is sold by the *board foot* (bd. ft.).

A board foot is a board a foot square, or its equivalent, and one inch or less in thickness.

In measuring lumber the thickness is regarded only when it exceeds one inch.

A board 16 feet long, 1 foot wide, and 1 inch thick contains 16 board feet. If less than an inch thick, it is still reckoned as 16 board feet. If $1\frac{1}{2}$ inches thick, it contains $1\frac{1}{2}$ times 16 board feet, or 24 board feet.

The price of lumber is usually given for a thousand feet, and the thousand feet indicated by the letter M.

The number of board feet in a piece of lumber is found by multiplying its length in feet by its width in feet by its thickness in inches.

In solving problems, reckon to the nearest board foot.

1. How many board feet in a board 18 ft. long, 1 ft. wide, and $\frac{7}{8}$ of an inch thick?

2. A floor board is 12 ft. long, 8 in. wide, and $\frac{7}{8}$ of an inch thick. How many board feet does it contain?

3. At 6 cents a board foot, what must be paid for 10 boards, each 12 ft. by 6 in. by $\frac{7}{8}$ of an inch?

4. How many board feet in 12 boards 16 ft. by $1\frac{1}{2}$ ft.? If the thickness is not mentioned, it is always understood to be 1 inch or less.

5. Allowing $\frac{1}{4}$ for waste, how many feet of flooring boards must be bought for a room 12 ft. by 10 ft.?

6. At $30 per M, what is the cost of 500 ft. of pine sheathing? 250 ft.? 200 ft.? 100 ft.? 800 ft.?

7. What is the rate per M, when hemlock boards are sold at 2 cents per board foot? $2\frac{1}{2}$ cents? $3\frac{1}{4}$ cents?

8. What is the rate per board foot when lumber is quoted at $30 per M? At $35? At $42?

9. Find the cost of 380 ft. of lumber at $45 per M.

10. What is the rate per M, when $97 is paid for 275 ft. of spruce boards?

At \$45 per M, what is the cost of 24 boards each 16 ft. long, 1 ft. wide, and $1\frac{1}{4}$ in. thick?

(1) $24 \times 16 \times 1 \times \frac{5}{4} = 480$, the number of board feet
480 bd. ft. = .48 of a thousand feet
$.48 \times \$45 = \$21.60$

(2) $24 \times 16 \times 1 \times \frac{5}{4} = 480$, the number of board feet
$\$45 \div 1000 = \$0.045$, the cost of a board foot
$480 \times \$0.045 = \$21.60$

11. What is the cost of 20 planks $16' \times 9'' \times 2\frac{1}{2}''$ at $60 per M?

12. At $36 per M, what is the value of 60 boards $10' \times 10'' \times \frac{7}{8}''$?

13. 24 sticks of timber 12 ft. long and 8 in. square were bought at $32 per M. What was the cost?

14. What is the cost of 30 joists, $3'' \times 4''$, 16 ft. long at $28 per M?

15. How many feet, board measure, in a chopping block 2 ft. 6 in. high and covering a space of 4 square feet on the ground? Find its value at $24 per M.

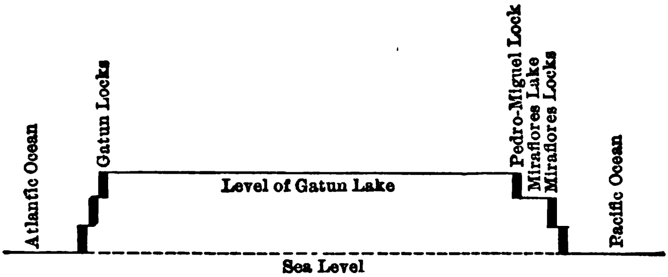
16. At $30 per M, how much will it cost to build on the baseball grounds a catcher's fence 12 ft. long and 6 ft. high, supported by four $3'' \times 4''$ joists set 2 ft. in the ground?

PROBLEMS ABOUT THE PANAMA CANAL

1. The Panama Canal Zone is a strip of land 10 miles wide across the Isthmus of Panama, $47\frac{2}{5}$ miles. What is the area of the zone? (Keep answers.)

2. It was ceded to the United States in February, 1904. How long ago? (Give answer in years and months.)

3. The total length of the canal is about 50 miles, 15 of which are at sea level. What part of the canal is above sea level?



PROFILE OF PANAMA CANAL

4. Vessels entering on the Atlantic side will be raised to the level of Gatun lake by three locks, each having a lift of 28 feet 4 inches. How many feet above sea level is Gatun lake?

5. Gatun lake is about 32 miles long. This is what part of the whole canal? Of the canal above sea level?

6. After crossing Gatun lake, vessels will be lowered by the Pedro-Miguel locks $\frac{6}{17}$ of the height above sea level to Miraflores lake. How many feet above sea level is Miraflores lake?

7. The distance across Miraflores lake to Miraflores locks is about $1\frac{2}{5}$ miles. How many feet?

8. At Miraflores vessels will be lowered to sea level by two locks equal in height. How many feet by each lock?

9. Each of the six locks is 1000 feet long and 110 feet wide, inside dimensions. The length is how many times the width? Each lock is what part of a mile in length? What are the dimensions of each lock in rods? The area in acres?

10. The crest of the great dam at Gatun is 10 yards above the height of Gatun lake. How many feet above sea level?

11. The dam at Gatun is 9040 feet long and 100 feet wide at its crest. What is the ratio of the length to the width? The length in miles? The width in rods? The area in acres?

12. The ocean steamship *Mauretania* is 790 feet in length and 88 feet in width. How many feet less than the length and the width of one of the locks?

13. The distance from New York to San Francisco by way of Straits of Magellan is 13,714 miles; by way of the canal the distance will be 5299 miles. How many miles will be saved by the Panama route?

14. The commercial route from Liverpool to San Francisco is 14,084 miles; by the canal it will be 8038 miles. How many miles shorter by the canal?

15. In going from New Orleans to Adelaide, Australia, a vessel sails 13,543 miles; 3209 miles will be saved by the canal. How far from New Orleans to Adelaide by way of the canal?

16. Calling the distance from Liverpool to San Francisco by way of Straits of Magellan 14,000 miles, and by way of the canal 8000 miles, and the average speed of a steamship 20 miles an hour, find:

(*a*) The number of days required to make the trip by way of Straits of Magellan.

(*b*) The number of days required to make the trip by way of the canal.

(*c*) The number of days saved by the canal route. Find this in two ways.

17. The estimated cost of the canal is $375,000,000. What is the average cost per mile?

18. The Suez canal is a sea-level canal 90 miles long and cost $100,000,000. Find the average cost per mile.

19. The cost of the Panama canal is how much more per mile than the cost of the Suez canal per mile?

AVOIRDUPOIS WEIGHT

1. How many ounces in 1 pound?

2. How many pounds in an ordinary ton?

3. Some articles are sold by the hundredweight (cwt.). How many hundredweight make 1 ton?

4. A long ton contains 240 pounds more than the ordinary ton. How many pounds make a long ton?

In problem work always use the ordinary ton unless otherwise directed.

5. Write the table of Avoirdupois weight.

6. What is the ratio of a pound to a hundredweight? A hundredweight to a ton? A hundredweight to a long ton? An ordinary ton to a long ton?

7. At 32 cents a pound, how much will 1 pound 14 ounces of butter cost?

8. When 50 pounds of ice are sold for 15 cents, what is the rate per ton?

9. How many pound packages can be put up from $2\frac{1}{2}$ hundredweight of cereal?

10. When a bushel of oats (32 pounds) costs 50 cents, how much will a ton cost?

11. An ice dealer pays 6 cents per hundredweight for his ice. At what rate per hundredweight must he retail it to make $1.50 per ton if the expense of handling is $1.30 per ton?

12. Find the cost of 3800 pounds of coal at $6.50 per ton.

13. Find the cost of 2800 pounds of fertilizer at $36 per ton, and 1600 pounds at $37.50 per ton.

14. A dealer in farm supplies bought 75 long tons of fertilizer and sold it by the ordinary ton. How many tons did he sell?

15. A shipment of wheat weighed 224,000 pounds. How many ordinary tons? How many long tons?

16. Mr. Evans sold the following quantities of shorts: $\frac{7}{8}$ of a ton, 25 hundredweight, $1\frac{3}{4}$ tons, 1500 pounds, 1200 pounds. How many tons?

17. Out of a consignment of 2 tons of potatoes, 32 hundredweight were sold. What part of a ton was left?

18. Find the cost of 2780 pounds of oat straw at 85 cents per hundredweight.

19. Mr. Wheeler paid $35 for $2\frac{1}{2}$ tons of shorts. What was the price per hundredweight?

20. Rye meal is 2.25 cents per pound. Find the cost of 3.6 hundredweight.

21. $36.45 was paid for oats at 45 cents a bushel. Find the weight, reckoning 32 pounds to a bushel.

22. Mr. Norcross sold 792 pounds of barley, 48 pounds to the bushel, at 90 cents a bushel. Find how much he received for it.

23. The equipment of a 25-acre hay farm in South Carolina, as given in a report by the Department of Agriculture, was: 1 Georgia plow stock with attachments, \$5; 1 smoothing harrow, \$10; 1 weeder, \$7; 1 curved-tooth and slicing harrow, \$10; 1 mower, \$35; 1 hayrake, \$15; 1 wagon, \$30; 1 hay press, \$90; 1 wooden roller, \$7.50. Total cost?

24. The expenses and receipts of this farm for one year:

EXPENSES

One man at \$15 per month	\$
One horse, cost of keeping at \$8.50 per month	
Interest on 25 acres of land, valued at \$100 per acre, at 8%	
Depreciation on \$209.50 worth of machinery, at 20%	
Total expense	\$

RECEIPTS

7 acres crimson clover hay, 3500 pounds per acre, at \$1.25 per hundredweight	\$
18 acres oat hay, 3000 pounds per acre, at \$25 per ton	
22 acres cowpea hay, 1½ tons per acre, at \$1.25 per hundredweight	
3 acres corn, 66⅔ bushels per acre, at 75 cents per bushel	
3 acres corn stover, 4000 pounds per acre, at \$15 per ton	
Total income	\$

Find (1) the total expense; (2) the total income; (3) the net income.

LIQUID MEASURE

1. Repeat the table of liquid measure.

2. How many pints in 1 gallon? 1 gallon 1 quart? 1 gallon 1 quart 1 pint?

3. How many quarts in $6\frac{3}{4}$ gallons? How many pints?

4. How many gallons and quarts of vinegar in a 5-gallon jug one half full?

5. How many half-pint bottles can be filled from $2\frac{1}{2}$ quarts of cream?

6. Four half-pint bottles of cream cost 1 dollar. The milkman allows 10 cents for each bottle returned. What is paid a quart for cream?

7. Mrs. Mason bought a gallon of maple sirup. She sold 2 quarts 1 pint to a neighbor. What part of the gallon did she sell?

8. If Mrs. Mason sold the sirup at cost, $1.20 per gallon, how much did she receive for what she sold?

CONVENIENT EQUIVALENTS

1 gallon = 231 cubic inches
1 cubic foot = $7\frac{1}{2}$ gallons (nearly)

9. What is the capacity in gallons of a tank 24 inches long, 11 inches wide, and 14 inches deep?

$$\frac{24 \times 11 \times 14}{231}$$

$24 \times 11 \times 14$ represents what?

$\frac{24 \times 11 \times 14}{231}$ represents what?

Find the capacity in gallons of tanks of these dimensions:

	LENGTH	WIDTH	DEPTH		LENGTH	WIDTH	DEPTH
10.	22″	21″	9″	11.	22″	14″	6″
12.	30″	28″	22″	13.	44″	28″	27″

	LENGTH	WIDTH	DEPTH		LENGTH	WIDTH	DEPTH
14.	2′9″	1′9″	1′3″	**15.**	2′9″	2′3″	2′6″
16.	4′	3′	2′	**17.**	5′	3′	4′
18.	4′	3½′	5½′	**19.**	3⅔′	3½′	4⅔′

20. A tin dish 11 inches long, 7 inches wide, and 3 inches deep holds how many gallons? How many if twice as long? If twice as long and twice as wide? If twice as long, twice as wide, and twice as deep?

21. A tank 28 inches long, 22 inches wide, and 18 inches deep is full of water. How many gallons?

22. How many gallon jugs can be filled from a tank 7 feet long, 3 feet wide, and 5½ feet deep?

23. A hot-water tank on the back of a stove is 2 feet 4 inches long, 1 foot 2 inches wide, and 11 inches deep. How many gallons does it hold?

24. A copper boiler covers 168 square inches on the top of a stove. It stands 22 inches high. What is its capacity in gallons?

25. How many times will you have to fill and empty a gallon dipper with water to fill a tank 1 foot 3 inches long, 1 foot 2 inches wide, and 11 inches deep?

26. A watering trough is 2 feet 6 inches long, 2 feet 4 inches wide, and 1 foot 10 inches deep. How many gallons of water will it hold?

27. The capacity of an oil tank is 252 cubic feet. What is its capacity in barrels of 42 gallons each?

DRY MEASURE *Oral*

1. Repeat the table of dry measure.

2. A quart is what part of a peck? A peck of a bushel? A quart of a bushel?

3. Express as pecks: 4 bu. $2\frac{3}{4}$ bu. 40 qt. 37 qt.

4. Express as bushels: 48 pk. 27 pk. 64 qt. 70 qt.

5. Express as quarts: 24 pk. 27 pk. 2 bu. 1 bu., 1 pk.

6. A quart of barley weighs $1\frac{1}{2}$ pounds. What is the weight of a peck? A bushel?

7. A bushel of wheat weighs 60 pounds. A peck weighs —— pounds. A quart weighs —— pound.

CONVENIENT EQUIVALENTS

1 bushel = 2150.42 cubic inches = $1\frac{1}{4}$ cubic feet (nearly).

Small fruits, beans, oats, wheat, shelled corn, etc., are measured by the stricken bushel, that is, a bushel measure even full, $1\frac{1}{4}$ cubic feet (nearly).

Larger fruits, potatoes, apples, corn in the ear, etc., are measured by the heaped bushel, $1\frac{1}{2}$ cubic feet (nearly).

WRITTEN PROBLEMS

1. What is the capacity in bushels of a bin 16 feet long, 7 feet wide, and 5 feet deep?

$(16 \times 7 \times 5) \div 1\frac{1}{4}$ 16 × 7 × 5 represents what?

$16 \times 7 \times \cancel{5} \times \frac{4}{\cancel{5}} = 448$ $(16 \times 7 \times 5) \div 1\frac{1}{4}$ represents what?

Find the capacity in bushels of bins of these dimensions:

	LENGTH	WIDTH	DEPTH		LENGTH	WIDTH	DEPTH
2.	7′	5′	3′	**3.**	10′	8′	5′
4.	15′	5′	4′	**5.**	20′	6′	8′
6.	5′	4′ 8″	2′ 8″	**7.**	4′	$3\frac{1}{3}$′	3′
8.	6′ 8″	3′ 9″	5′	**9.**	16′	4.5′	2.25′

10. A box 2½ feet by 2½ feet by 2½ feet is full of meal. How many bushels?

11. A farmer's corn crib is 8 feet by 2½ feet by 6 feet. How many bushels will it hold?

12. At 32 pounds to a bushel, what is the weight of oats in a box 5 feet by 2 feet by 3 feet?

13. A corn crib, 12 feet by 4 feet by 5 feet, is filled with corn in the ear. It takes 2 bushels of corn in the ear to make 1 bushel of shelled corn. The corn in the crib will make how many bushels of shelled corn?

14. Reckoning 56 pounds to a bushel, what is the weight of the corn in a box 2½ feet long, 2 feet wide, and 1½ feet deep?

15. At 98 cents a bushel, what ought Mr Akers to receive for the wheat that just fills a bin 8 feet by 4 feet by 2½ feet?

16. Mr. Edwards has in his cellar a bin 15 feet by 10 feet by 6 feet, filled with apples. How many bushels (1½ cubic feet to a bushel)?

SCHOOL KITCHEN PROBLEMS

1. Find the total cost of the following cooking outfit for a country school: 1 blue-flame, 3-burner, kerosene stove, $6.25; 2 dish pans @ 25¢; 2 salt and 2 pepper castors @ 10¢; 10 rolling pins @ 15¢; 10 teacups and 10 saucers @ 5¢; 10 plates @ 8¢; 3 agate saucepans @ 35¢; 10 wooden spoons @ 10¢; 1 dozen knives, $1.50; 1 dozen forks, $1.50; 1 dozen spoons @ 10¢; 10 vegetable knives @ 15¢; 5 strainers @ 10¢; 10 beaters @ 15¢; 10 measuring cups @ 8¢; 10 tin plates @ 5¢;

13 bowls @ 5¢; 10 custard cups @ 8¢; 6 bread tins @ 10¢; 1 biscuit tin 10¢; 1 potato masher 10¢; 1 flour sifter 15¢.

MENU OF DINNER SERVED TO SIX PERSONS BY THE GIRLS IN THE SCHOOL IN PROBLEM 1

Menu

SOUP

TOMATO BISQUE CROUTONS

RELISHES

CELERY PICKLES

ROAST

SIRLOIN OF BEEF

VEGETABLES

RICED POTATOES PICKLED BEETS

SALAD

WALDORF WAFERS

DESSERT

CUSTARD, WITH CARAMEL SAUCE

GRAPES MILK

2. Using the tables below, find the cost of each course; of the entire dinner; for each person.

COST OF MATERIAL

Apples, 15¢ per doz.
Beef, 25¢ per lb.
Beets, 6¢ per lb.
Bread, 5¢ per loaf
Butter, 32¢ per lb.
Celery, 15¢ per bunch
Eggs, 35¢ per doz.
Grapes, 2 lb. for 25¢
Lettuce, 7¢ per head

Mayonnaise dressing, 30¢
Milk, 7¢ per qt.
Pickles, 10¢ per qt.
Potatoes, 24¢ per pk.
Sugar, 6¢ per lb.
Tomatoes, 16¢ per can
Wafers, 15¢ per box
Flour and seasonings for Bisque, 3¢

Quantities used:

(1) *Soup:* ½ can tomatoes, 1 qt. milk, 3 tbsp. flour, 3 tbsp. butter (1½ oz.), seasonings (onion, salt, cloves, pepper, celery, parsley, bay-leaf, sugar, soda). One loaf bread for croutons and dinner.

Relishes: 1 bunch celery, 1 pt. pickles.

(2) *Roast:* 5 lb. beef.

Vegetables: 1 qt. potatoes, ¼ cup butter (2 oz.), 1 lb. beets.

(3) *Salad:* 4 apples, ⅔ bunch celery, 1 head lettuce, Mayonnaise dressing. Wafers, 1 box.

(4) *Dessert:* Custard: 6 eggs, 1 qt. milk, 8 tbsp. sugar (4 oz.).
Caramel sauce: 1½ cups sugar (¾ lb.).
Grapes: 2 lb. Milk: 2 qt.

3. Find the cost of a similar dinner served to 20 persons.

TIME MEASURE

The unit of time measure is the day, or 24 hours, the time required for a complete rotation of the earth on its axis.

TABLE OF TIME MEASURE

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
30 days = 1 commercial month
28, 29, 30, or 31 days = 1 calendar month
12 calendar months = 1 year (yr.)
365 days = 1 year
366 days = 1 leap year
10 years = 1 decade
100 years = 1 century

1. The minute hand of a clock moves how many times as fast as the hour hand?

2. A clock that ticks seconds will tick how many times in an hour? In a day?

3. A clock striking hours will strike how many times in a day?

4. The speedometer of an automobile shows a rate of 24 miles per hour. What is the rate per minute?

5. An automobile runs a mile in $2\frac{2}{5}$ minutes. What rate per hour does the speedometer show?

6. Two clocks show the same time at noon to-day. One runs perfectly, and the other gains $\frac{1}{6}$ of a minute every hour. What time will the latter show at noon to-morrow?

7. Ashton's watch loses 10 seconds an hour. If exactly right at 9 o'clock in the morning on Tuesday, what is the correct time on Thursday afternoon when his watch says it is 3 o'clock?

8. Louise left home at 2:47 o'clock and arrived at her uncle's at 5:28. How many hours and minutes was she on the way?

9. It took Harry 3 hours 42 minutes to return from the summer camp. He left camp at 10:53 o'clock in the forenoon. At what time did he reach home?

10. At a rate of 18 miles an hour, it takes a touring car 1 hour and 5 minutes to go from A to B, 15 minutes being spent in replacing a broken tire. How many miles apart are the two places?

11. Years divisible by 4 are leap years, except centennial years not divisible by 400. Tell which of these years are leap years: 1912; 1918; 1920; 1900; 2000.

DRILL EXERCISES

Change to the next higher denomination:

1. 400 min.
2. 27 pt.
3. 120 oz.
4. 602 in.
5. 3456 sq. in.
6. 228 sec.
7. 24,600 lb.
8. 16,400 qt. (dry)
9. 1452 doz.
10. 1089 cu. ft.
11. 640 cu. ft. (wood)
12. 769 cd. ft.
13. 1680 rd.
14. 302 qt.
15. 200 hr.
16. 242 sq. ft.
17. 378 ft.
18. 5000 cu. in.
19. 10,587¢
20. 692 yd.
21. 302 qt. (liquid)
22. 150 da.
23. 1010 reams
24. 7680 doz.

Change to the next lower denomination:

1. $\frac{7}{8}$ mi.
2. $\frac{5}{8}$ bu.
3. $\frac{3}{5}$ A.
4. $\frac{1}{9}$ wk.
5. $\frac{1}{8}$ ream
6. $\frac{7}{16}$ cd.
7. $\frac{19}{40}$ da.
8. $\frac{1}{5}$ cu. yd.
9. $\frac{1}{2}$ sq. yd.
10. \$$\frac{3}{8}$
11. $\frac{9}{20}$ gal.
12. $\frac{5}{16}$ T.
13. $\frac{9}{11}$ yr.
14. $\frac{5}{6}$ sq. ft.
15. $\frac{7}{16}$ cu. ft.
16. $\frac{1}{11}$ rd.
17. $\frac{5}{16}$ hr.
18. $\frac{7}{10}$ cwt.

Find the value of:

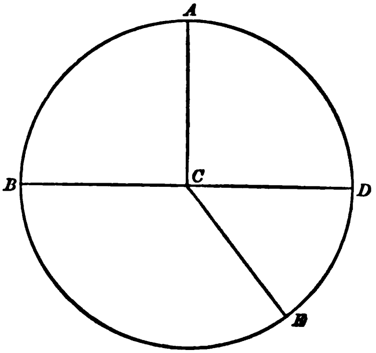
1. .27 mi.
2. .625 bu.
3. .325 A.
4. .03 wk.
5. .09 ream
6. .33 cd.
7. .905 da.
8. .4 cu. yd.
9. .6 sq. yd.
10. \$0.675
11. .48 gal.
12. .54 T.
13. .095 yr.
14. .22 sq. ft.
15. .65 cu. ft.
16. .015 rd.
17. .35 hr.
18. .875 yd.

Change:

1. 3 da. 12 hr. to min.
2. 24 rd. 3 yd. to yd.
3. $4\frac{1}{4}$ T. 15 lb. to lb.
4. 3 cd. 56 cu. ft. to cu. ft.
5. 24 rd. 12 ft. to ft.
6. 72 bu. 3 qt. to qt.
7. 7 pk. 3 qt. to qt.
8. 11 gal. 2 qt. to qt.
9. 15 A. 72 sq rd. to sq. rd.
10. 10 reams 7 qr. to qr.
11. 3 mi. 100 rd. to ft.
12. 5 yd. 2 ft. to ft.
13. 3 bu. 3 pk. 3 qt. to qt.
14. 3 A. 3 sq. rd. to sq. rd.
15. 3 sq. rd. 30 sq. ft. to sq. ft.
16. 4 mi. 6 ft. to in.
17. 15 cd. to cu. ft.
18. $1\frac{7}{12}$ da. to hr.
19. $8\frac{3}{4}$ gal. to qt.
20. $15\frac{3}{4}$ bu. to qt.

CIRCLES

A *circle* is a surface bounded by a curved line every point of which is equally distant from a point within called the *center*. The boundary line is the *circumference*.



A *diameter* of a circle is a straight line running through the center and terminating at the circumference on either side. *BD* is a diameter of the circle above.

A *radius* is a straight line running from the center to the circumference. *CA*, *CD*, *CB*, are radii of the circle above.

Compare the radius and the diameter.

A portion of a circle inclosed by two radii and a part of the circumference is a *sector*. *CDE* is a sector of the circle on page 228.

Using a tape measure, find the circumference and the diameter of any circular object. Divide the length of the circumference by the length of the diameter, carrying out the division three decimal places. How many times is the diameter contained in the circumference?

By very accurate measurements it has been found that the circumference is 3.14159+, or *3.1416* times the diameter. Since 3.1416 equals 3$\frac{1}{7}$, nearly, for approximate measurements the circumference may be regarded as 3$\frac{1}{7}$ times the diameter.

FINDING THE CIRCUMFERENCE

Using 3$\frac{1}{7}$ as the ratio of the circumference to the diameter, find the circumference of:

1. A plate 7 inches in diameter.

2. A carriage wheel 35 inches in diameter.

3. A circular table whose diameter is 28 inches.

4. A pencil whose diameter is $\frac{7}{22}$ of an inch.

5. A circle drawn with a 21-inch radius.

6. A circular garden plot whose radius is 4$\frac{3}{8}$ feet.

7. A 50-cent piece.

8. Tell how to find the approximate circumference of a circle when the diameter is given; when the radius is given.

FINDING THE DIAMETER AND THE RADIUS

Find the approximate diameter and radius of:

1. A bicycle wheel whose circumference is 88 inches.

2. A stove lid 22 inches in circumference.

3. A circular pond whose shore line is 242 rods.

4. A water main 3⅔ feet in circumference.

5. A carriage wheel covering a distance of 198 inches in one revolution.

6. A magnifying glass the distance around whose rim is 2½ inches.

7. Tell how to find the approximate diameter of a circle when the circumference is given; how to find the radius when the circumference is given.

PROBLEMS

In solving the following problems call the circumference 3.1416 times the diameter.

1. What is the circumference of a circle drawn with a 20-inch radius?

2. The shore line of a circular pond is 7854 feet. How far is it across the pond in a straight line?

3. The distance around an old oak tree is 94.248 inches. What is the distance through the tree?

4. A circular flower bed has a radius of 8 feet. What is its circumference?

5. The distance around a circular grass plot is 471.24 feet. What is its diameter?

6. A circular race track is 2640 feet long. What length of radius was used to lay out the track?

7. A baseball is 2.875 inches in diameter. What is its circumference?

8. What is the circumference of the largest circle you can draw on a piece of paper 7 inches by 10 inches?

In solving the following problems call the circumference $3\frac{1}{7}$ times the diameter.

9. A bicycle wheel is 28 inches in diameter. How many inches does it go in one revolution? How many feet in 1000 revolutions? How many miles?

10. How many revolutions will be made by a bicycle wheel 28 inches in diameter in going 1 mile?

11. A wagon wheel has a radius of 21 inches. How long a piece of iron must be used for the tire?

12. What must be the diameter of a chimney hole to receive a stovepipe 21 inches in circumference?

13. A man on the shore of a circular pond wishes to go to a point on the opposite side. The distance by boat is 35 rods. Being afraid to go by boat, he walks along the shore. How far does he walk? What part of a mile is it by boat? By walking?

Calling the ratio of the circumference to the diameter 3.1416, find the missing dimensions:

	RADIUS	DIAMETER	CIRCUMFERENCE
14.	4 feet		?
15.		?	20 feet
16.	?		17 rods

AREA OF CIRCLE

Draw a large circle and divide it into a large number of equal sectors. Cut into semicircles. Cut each semicircle along the line dividing it into sectors. Arrange the parts of one semicircle as in the lower part of figure 2. Arrange the parts of the other semicircle as in the upper part of figure 2. The figure produced is practically a parallelogram whose base is $\frac{1}{2}$ of the circumference of the circle and whose altitude is the radius of the circle.

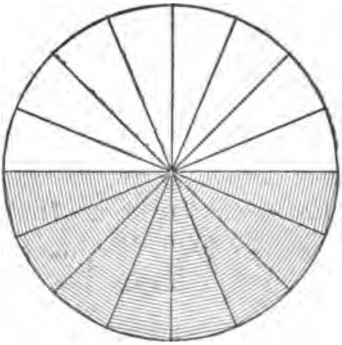


FIGURE 1

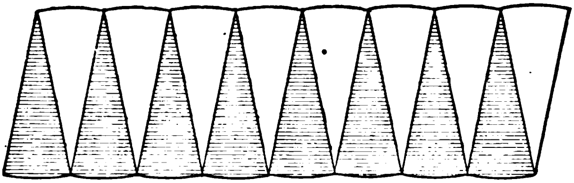


FIGURE 2

The area of a circle is equal to one half the product of its circumference and its radius.

Calling the circumference $3\frac{1}{7}$ times the diameter, find the area of:

1. A plate 7 inches in diameter.

2. A circular table 28 inches across.

3. A piano stool 14 inches in diameter.

4. A circular bed of pansies, if the bed is 35 inches in diameter.

5. A circle drawn with a 3½-foot radius.

6. A button 1⅗ inches in diameter.

7. A medal whose circumference is 22 inches.

8. A circular window the distance around whose edge is 5 feet 6 inches.

9. A watch crystal 1¾ inches in diameter.

10. A 50-cent piece.

Find the exact area of:

1. The base of a tin pail whose diameter is 8 inches.

2. The end of a log 15 inches in diameter.

3. The surface covered by the bottom of a pitcher 5 inches across.

4. A clock dial whose radius is 4 inches.

5. The largest circle that can be drawn on a piece of paper 7.5 inches square.

PROBLEMS

1. Find the area over which a cow can feed if tied to a stake by a rope 28 feet long. (Use 3⅐.)

2. The curbing about a circular pond is 78.54 feet. Find the area covered by the pond. (Use 3.1416.)

3. The bottom of a barrel is 1 foot 9 inches in diameter. What area does it cover on the floor? (Use 3⅐.)

4. What is the area covered by the base of a circular kettle 15 inches in diameter? (Use 3.1416.)

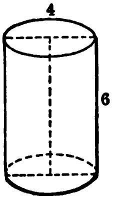
5. At $1\frac{1}{2}$ cents a foot, how much will it cost to fence a circular bed of tulips if the distance across the bed is 56 inches? (Use $3\frac{1}{7}$.)

6. A farmer has a well $3\frac{1}{2}$ feet in diameter. What is the area of the opening? (Use $3\frac{1}{7}$.)

7. Find the approximate area of the base of a circular chimney 35 feet in diameter; the exact area.

CYLINDER: SURFACE, VOLUME

Of what shape are the ends or bases of this solid? How do they compare in size? If the bases are equal and parallel, what is true of their diameters? What is true of the diameter of the whole solid?

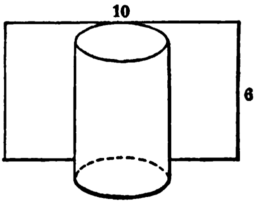


A solid of uniform diameter whose bases are equal and parallel circles is a *cylinder*.

The *altitude* of a cylinder is the perpendicular distance between its bases.

The lateral or curved surface of a cylinder is the *convex* surface.

Exactly fit a piece of paper around a cylinder. What form has it when unrolled? How does the length of the rectangle compare with the circumference of the base? How does the width of the rectangle compare with the altitude of the cylinder? How can the convex surface of a cylinder be found?



The convex surface of a cylinder is equal to the product of its circumference and its altitude.

The entire surface of a cylinder is equal to the sum of the convex surface and the areas of the bases.

1. What is the convex surface of a cylinder whose circumference is 14 in. and whose altitude is 3 ft.?

Find the convex surface of these cylinders:

	DIAMETER	HEIGHT		DIAMETER	HEIGHT
2.	10 in.	15 in.	**3.**	15 in.	18 in.
4.	25 in.	35 in.	**5.**	12 in.	45 in.
6.	4 ft.	12 ft.	**7.**	6 ft.	16 ft.

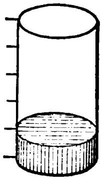
8. Find the entire surface of the above cylinders.

9. What is the convex surface of a stovepipe 7 in. in diameter and 3 ft. long?

10. A tin dish is 5 in. in diameter and 10 in. high. What is its entire outside surface?

11. What is the entire outside surface of a log 25 in. in diameter and 14 ft. long?

12. At 12¢ a square yard, how much will it cost to paint the convex surface of a silo 15 ft. in diameter and 24 ft. high?



If the base of this cylinder covers 2 sq. in., how many cubic inches are represented by a layer 1 in. high? How many cubic inches in the whole cylinder?

Tell how to find the volume of a cylinder.

The volume of a cylinder is found by multiplying the area of its base by its altitude.

Find the volumes of these cylinders:

	DIAMETER	HEIGHT		DIAMETER	HEIGHT
1.	21 in.	49 in.	2.	2 ft. 4 in.	4 ft.
3.	3½ ft.	7 ft.	4.	6.25 ft.	8.5 ft.
5.	8 ft.	15 ft.	6.	6 ft.	14 ft.

7. Mr. Merriam has a silo 17.5 ft. in diameter and 25 ft. high. What is its capacity in cubic feet?

8. How many gallons of water will be required to fill a kettle whose diameter is 14 in. and whose height is 12 in.?

9. A water tank by the side of a railroad track is 12 ft. in diameter and 15 ft. high. Find its capacity in gallons.

10. A copper hot water boiler is 15 in. in diameter and 4 ft. high. How many square feet of copper were required to make it? What is the capacity of the boiler in gallons?

11. An iron ash can whose base covers 3½ sq. ft. is 2 ft. 4 in. deep. Find its capacity.

PERCENTAGE

Oral

The expression per cent means by hundredths.

Thus, 3 per cent of a dollar means 3 hundredths, or .03, of a dollar.

The sign % means the same as the expression per cent. Thus, 7 % of a dollar and 7 per cent of a dollar mean the same thing — 7 hundredths, or .07, of a dollar.

1. Into how many hundredths can a dollar be separated? A ton of coal? A yard of cloth? Anything?

2. What per cent of a dollar is 1 cent? 10 cents? 25 cents? 50 cents? 75 cents? 80 cents? 100 cents?

3. What part of a whole thing is 1 % of it? 10 % of it? 25 % of it? 50 % of it? 75 % of it? 100 % of it?

4. What per cent of a whole thing is $\frac{1}{2}$ of it? $\frac{1}{4}$ of it? $\frac{1}{5}$ of it? $\frac{1}{10}$ of it? $\frac{1}{20}$ of it? $\frac{1}{50}$ of it? $\frac{1}{100}$ of it? $\frac{100}{100}$ of it?

5. A man bequeathed 60 % of his estate to his family, and the balance equally to four charitable institutions. What per cent of the estate to each institution?

6. A dealer in farm supplies sold $\frac{3}{5}$ of his stock of seeds in the spring, 25 % in the summer, and the remaining part in the fall. What per cent in the fall?

PER CENT AND FRACTIONAL EQUIVALENTS

$2\% = \frac{1}{50}$
$4\% = \frac{1}{25}$
$5\% = \frac{1}{20}$
$6\frac{1}{4}\% = \frac{1}{16}$
$8\frac{1}{3}\% = \frac{1}{12}$
$10\% = \frac{1}{10}$
$12\frac{1}{2}\% = \frac{1}{8}$
$16\frac{2}{3}\% = \frac{1}{6}$

$20\% = \frac{1}{5}$
$25\% = \frac{1}{4}$
$30\% = \frac{3}{10}$
$33\frac{1}{3}\% = \frac{1}{3}$
$37\frac{1}{2}\% = \frac{3}{8}$
$40\% = \frac{2}{5}$
$50\% = \frac{1}{2}$
$60\% = \frac{3}{5}$

$62\frac{1}{2}\% = \frac{5}{8}$
$66\frac{2}{3}\% = \frac{2}{3}$
$70\% = \frac{7}{10}$
$75\% = \frac{3}{4}$
$80\% = \frac{4}{5}$
$83\frac{1}{3}\% = \frac{5}{6}$
$87\frac{1}{2}\% = \frac{7}{8}$
$90\% = \frac{9}{10}$

NOTE. This table must be memorized.

1. Subtract each of the per cents in the table from 100 %.

2. Subtract each fraction from 1 and express the result as per cent.

$125\% = \frac{125}{100} = 1.25.$

3. Express decimally:

120% $137\frac{1}{2}\%$ $166\frac{2}{3}\%$ 100% 105% $106\frac{1}{4}\%$

$150\% = \frac{150}{100} = \frac{3}{2}$.

4. Express as common fractions:

125% 140% $162\frac{1}{2}\%$ $116\frac{2}{3}\%$ $108\frac{1}{3}\%$ 105%

5. Express as per cents:

$\frac{3}{2}$ $\frac{4}{3}$ $\frac{5}{4}$ $\frac{6}{5}$ $\frac{7}{6}$ $\frac{9}{8}$ $\frac{11}{10}$ $\frac{13}{12}$ $\frac{23}{20}$ $\frac{27}{25}$ $\frac{59}{50}$

$\frac{1}{4}\% = \frac{1}{4}$ of $1\% = .00\frac{1}{4}$.

6. Express decimally:

$\frac{1}{2}\%$ $\frac{3}{4}\%$ $\frac{2}{5}\%$ $\frac{1}{6}\%$ $\frac{7}{8}\%$ $\frac{2}{3}\%$ $\frac{3}{10}\%$

$1\% = \frac{1}{100}$. $\frac{3}{5}\% = \frac{3}{5}$ of $\frac{1}{100} = \frac{3}{500}$.

7. Express as common fractions:

$\frac{1}{2}\%$ $\frac{1}{4}\%$ $\frac{2}{3}\%$ $\frac{1}{6}\%$ $\frac{4}{5}\%$ $\frac{1}{8}\%$ $\frac{1}{10}\%$ $\frac{1}{12}\%$

TERMS USED IN PERCENTAGE

There are three terms used in every problem in percentage:

(1) The Whole, or the Base.

(2) The Part, or the Percentage.

(3) The Value of the Part, or the Rate Per Cent.

The whole, or the base, is the number or quantity of which a certain number of hundredths is taken.

The part, or the percentage, is the part of the whole taken.

The value of the part, or the rate per cent, is the number of hundredths of the whole taken.

There are three classes of problems in percentage:

I. The whole and the value of the part given to find the part.

What is 25 % of \$80 ?

(1) $25\% = .25 = \frac{25}{100} = \frac{1}{4}$.
$\frac{1}{4}$ of \$80 = \$20.

$$\begin{array}{r} \$80 \\ \underline{.25} \\ \$20.00 \end{array}$$

II. The whole and the part given to find the value of the part.

\$20 is what per cent of \$80 ?

(1) $\frac{20}{80} = \frac{1}{4} = \frac{25}{100} = .25 = 25\%$.

(2) $$\begin{array}{r} .25 = 25\% \\ 80\overline{)20.00} \end{array}$$

III. The part and the value of the part given to find the whole.

\$20 is 25 % of what amount?

(1) \$20 is .25, or $\frac{25}{100}$, or $\frac{1}{4}$ of what?
\$20 is $\frac{1}{4}$; $\frac{4}{4}$, or the whole, is \$80.

(2) $$\begin{array}{r} \$80 \\ .25\overline{)\$20} = 25\overline{)\$2000} \end{array}$$

FINDING THE PART, OR THE PERCENTAGE *Oral*

Find :

1. 50 % of 84 bushels of corn ; 25 %.
2. 75 % of 80 acres of land ; 20 %.
3. 40 % of 60 quarts of milk ; 60 %.
4. 80 % of 120 bushels of wheat ; 10 %.
5. 30 % of 30 dollars ; 70 %.
6. 90 % of 160 dozen eggs ; $12\frac{1}{2}$ %.
7. $37\frac{1}{2}$ % of 56 tubs of butter ; $62\frac{1}{2}$ %.
8. $87\frac{1}{2}$ % of 48 cases of lard ; $33\frac{1}{3}$ %.
9. $66\frac{2}{3}$ % of 72 pounds of beef ; $16\frac{2}{3}$ %.
10. $83\frac{1}{3}$ % of 72 baskets of grapes ; $8\frac{1}{3}$ %.
11. $6\frac{1}{4}$ % of 80 pounds of coffee ; 5 %.

12. 4 % of 200 hours; $2\frac{1}{2}$ %.

13. $\frac{1}{2}$ % of 600 dollars; $\frac{2}{3}$ %.

Find $16\frac{2}{3}$ %; then 25 %; $33\frac{1}{3}$ %; 50 %; $66\frac{2}{3}$ %; 75 %; $83\frac{1}{3}$ % of:

14. \$12 \$48 \$84 \$24 \$108 \$132 \$240

15. \$36 \$60 \$96 \$72 \$120 \$144 \$300

16. What is $1\frac{1}{2}$ times \$40? What is 150 % of \$40?

Find:

17. 125 % of \$24; of \$36.

18. 120 % of \$30; of \$80.

19. $133\frac{1}{3}$ % of \$27; of \$45.

20. $116\frac{2}{3}$ % of \$60; of \$72.

21. 110 % of \$50; of \$90.

22. $112\frac{1}{2}$ % of \$80; of \$56.

23. $108\frac{1}{3}$ % of \$48; of \$60.

24. 105 % of \$60; of \$80.

25. What is 1 % of 400? What is $\frac{1}{2}$ % of 400?

Find:

26. $\frac{1}{3}$ % of \$600; of \$1200.

27. $\frac{1}{4}$ % of \$800; of \$3200.

28. $\frac{1}{5}$ % of \$1000; of \$4000.

29. $\frac{1}{6}$ % of \$1200; of \$7200.

30. $\frac{1}{7}$ % of \$2800; of \$5600.

31. $\frac{1}{8}$ % of \$2400; of \$5600.

32. $\frac{1}{9}$ % of \$3600; of \$6300.

33. $\frac{1}{10}$ % of \$4000; of \$9000.

DRILL EXERCISES

12	48	72
24	Find	96
132	50 %	120
108	of	144
36	60	84

In the center space use 25 %; 75 %; 33⅓ %; 66⅔ %; 16⅔ %; 83⅓ %; 8⅓ %; 125 %; 175 %; etc.

100	50	200
400	Find	150
250	10 %	350
450	of	500
550	600	300

In the center space use 2 %; 4 %; 5 %; 20 %; 25 %; 30 %; 40 %; 50 %; 60 %; 70 %; 75 %; 80 %; 90 %; 102 %; 104 %; etc.

16	48	80
32	Find	160
112	12½ %	96
144	of	176
128	64	192

In the center space use 37½ %; 62½ %; 87½ %; 25 %; 50 %; 75 %; 6¼ %; 137½ %; 162½ %; etc.

ORAL PROBLEMS

1. Three boys cleared the sidewalk of snow. What per cent of the pay should each get?

2. Mr. Howard cut 36 tons of hay and sold 75 % of it. How many tons did he sell?

3. Mr. Gordon raised 60 bushels of turnips and 20 % as many beans. How many bushels of beans?

4. Ward has 48 chickens and Kenneth 33⅓ % more. How many has Kenneth?

5. How many tons of fertilizer has a dealer left after selling 63⅔ % of a carload of 30 tons?

6. A cow that cost $56 was sold at a gain of 12½ %. What was the gain? What was the selling price?

7. How many days in 83⅓ % of the month of November? In 70 %? In 66⅔ %? In 50 %? In 60 %?

8. Stephen paid 96 cents for a geography and 62½ % as much for a grammar. How much for his grammar? For both?

9. Of $400 paid for repairs, the carpenter received 30 %, the roofer 25 %, the plumber 20 %, the painter 15 %, and the tinsmith 10 %. How much was paid to each?

10. Find:

8⅓ %	10 %	12½ %	16⅔ %	20 %	25 %
90 %	1. Of $120		6. Of 600 shovels		30 %
87½ %	2. Of $240 3. Of $2.40		7. Of 720 hoes 8. Of 840 rakes		33⅓ %
83⅓ %	4. Of $3.60		9. Of 960 scythes		37½ %
80 %	5. Of $4.80		10. Of 1200 forks		40 %
75 %	70 %	66⅔ %	62½ %	60 %	50 %

WRITTEN PROBLEMS

1. A farmer raised 325 chickens and sold 68 % of them. How many did he sell?

2. Mr. Fletcher hired a neighbor to cut his hay for 15 % of the crop. If the crop was 37 tons, how many tons did the neighbor get?

3. Mr. Leighton sold 75 % of his crop of 288 bushels of potatoes at 65 cents a bushel. How much did he receive for them?

4. A farmer has in his barn 650 bushels of grain, of which 46 % is corn, 38 % oats, and the remainder wheat. How many bushels of each kind of grain?

5. Of the 80 animals on a farm, 5 % are horses, 15 % cows, 35 % pigs, and 45 % sheep. How many animals of each kind?

6. A merchant having 576 tubs of butter sold 25 % of it to one grocer, 16⅔ % to another, 37½ % to a third, and 8⅓ % to a fourth. How many tubs to each grocer? How many tubs had he left?

7. A farmer raised 72 bushels of seed corn. After selling 37½ % to Mr. Bean and 33⅓ % to Mr. Holmes, how many bushels had he left?

8. In a civil service examination 76 % of the 175 applicants passed. How many passed?

9. By spraying his apple trees a farmer increased his crop 28 %. If he harvested 250 barrels in the year before spraying, how many barrels did he get the next year?

10. How many acres are left after 36 % of a farm of 350 acres is sold?

11. A man owing $168 paid 65 % of it in June and the rest in July. How much did he pay in June? In July?

12. Four per cent of a piece of cloth measuring 75 yards was spoiled in weaving. How many yards were good?

13. An ocean steamship sailed 542 miles on Tuesday and 85 % as many miles on Wednesday. How many miles on both days?

14. A merchant sold a lot of blankets for $270. Of this 24 % was profit. Find the profit.

15. Frank has a collection of 850 foreign postage stamps. How many will he have after selling 22 % of them?

16. On May 1 a merchant had in stock 1250 yards of gingham, and 825 yards of muslin. How many yards of each had he after selling 86 % of each kind?

17. Mr. Adams bought a flock of 225 sheep and sold 56 % of them to Mr. Martin. How many did Mr. Martin buy? How many had Mr. Adams left?

18. Mr. Watson has a pasture 80 rods by 64 rods, and a garden 6¼ % as large. How many acres in the pasture? In the garden?

19. A real estate dealer bought a lot of land containing 43,560 square feet. How many square feet had he after buying another lot containing 75 % as much?

20. A vessel lost 29 % of its cargo of 9500 bushels of wheat in a storm. How many bushels did it bring to port?

21. By the census of 1900 the population of a city was 12,375. By the census of 1910 it was 36 % more. What was the population in 1910?

FINDING THE VALUE OF THE PART, OR THE RATE PER CENT

Oral

1. What part of 100 is 1? 3? 9? 27? 51? 83? 100?

2. How many hundredths of 100 is 1? 3? 5? 7? 50? 82? 100?

3. What per cent of 100 is 1? 2? 5? 25? 62? 75? 100?

4. What part of 40 is 8? How many hundredths? What per cent?

5. What part of 24 is 12? Express your answer (1) as a common fraction; (2) as a decimal; (3) as a per cent.

6. In like manner express as parts of 24: 8 16 6 18 4 20 3 9 15 21 2

7. What per cent of $72 is $36? $24? $18? $54? $12? $60? $9? $27? $45? $63? $8? $6?

Express as per cents of:

8. A gallon: 2 quarts; 3 quarts; 1 pint; 1 quart 1 pint.

9. A bushel: 1 peck; 2 pecks; 3 pecks.

10. A peck: 1 quart; 3 quarts; 5 quarts; 6 quarts.

11. An inch: 6 inches; 8 inches; 9 inches; 10 inches.

12. A yard: 1 foot 6 inches; $\frac{3}{4}$ of a foot; $\frac{1}{2}$ of a foot; 2 feet 6 inches.

13. An acre: 40 square rods; 4 square rods; 100 square rods.

14. A mile: 40 rods; 80 rods; 200 rods; 160 rods; 32 rods.

15. Giving answers by groups, tell what per cent of $100 is:

$\$50$ $\$25$ $\$20$ $\$16\frac{2}{3}$ $\$12\frac{1}{2}$ $\$166\frac{2}{3}$
$\$100$ $\$125$ $\$120$ $\$116\frac{2}{3}$ $\$137\frac{1}{2}$ $\$187\frac{1}{2}$
$\$150$ $\$175$ $\$180$ $\$133\frac{1}{3}$ $\$162\frac{1}{2}$ $\$200$

16. Express as per cents of 4: 4 5 6 7 8

17. What per cent of 24 is 12? 24? 32? 40? 30? 42? 28? 44? 27? 33? 39? 45? 48?

30	20	40	15	45	12	24	36	48	10
60	What per cent of **60** is								50
57	39	27	21	9	3	54	42	18	6

Use 30 in the center space. Use 90.

ORAL PROBLEMS

1. What per cent of his money does Arthur save if he puts in the school savings bank 1 cent out of every 10 that he earns? If he puts in 2 cents? 4 cents? 5 cents?

2. Maria earns 25 cents each week. She saves 3 cents out of every 5 that she earns. What per cent does she save? How much does she save?

3. Out of 20 sessions of school Caroline was absent 1. What was her per cent of attendance?

4. Mr. Sampson planted his 24-acre lot as follows: 12 acres to corn, 8 acres to potatoes, and 4 acres to cabbages. What per cent of the lot was planted to each?

5. Emma and Winifred played 12 games of croquet. Emma won 4, and Winifred the rest. What per cent of the games did Winifred win?

6. What is the standing in per cent of a baseball team that loses 9 of the 24 games played?

7. Mr. Magoon's cow gives 8 quarts of milk in the morning and 12 quarts at night. The night's milking is what per cent of the whole quantity?

8. A grocer bought a 48-gallon barrel of kerosene and sold 16 gallons at once. What per cent of the barrel had he left?

9. Of the 27 votes cast by the common council for superintendent of streets, 18 were cast for Mr. Hume. What per cent of the members voted for Mr. Hume?

10. In the sixth grade there are 30 girls and 20 boys. What per cent of the pupils are girls? Boys?

11. The teacher gave 4 examples in arithmetic on Monday, 5 on Tuesday, 6 on Wednesday, and 10 on Thursday. The number given each day was what per cent of the total number given?

12. Alice solved all but 1 of the 25 examples given to the class. What per cent did she solve? Sherman solved 20. What per cent did he fail? Clara solved 22. What per cent?

13. In making a piece of machinery, one man worked 4 hours, another 6 hours, a third 8 hours, a fourth 12 hours, and a fifth 18 hours. What per cent of the work was performed by each?

WRITTEN PROBLEMS

1. A farmer sold 56 of his 175 sheep. What per cent of his flock did he sell?

2. From a case of cereal (144 packages) there were sold 108 packages. What per cent?

3. Ninety-nine of the 220 pages of an arithmetic are given up to drill exercises. The drill exercises are what per cent of the book?

4. Fred had a kite string 750 feet long and lost 270 feet. What per cent?

5. A merchant having 180 chests of tea sold 153 of them. What per cent of his stock did he sell?

6. A dry-goods dealer paid $228 for newspaper advertising and $78 for circulars. How much did his advertising cost? What per cent of this was paid for advertising in newspapers? What per cent for advertising by circulars?

7. In draining his land, Mr. Dean used 406 feet of pipe in one field and 294 feet in another. What per cent of the pipe used was laid in each field?

8. In a test at an agricultural station it was found that 132 of the 150 beet seeds planted produced plants. What per cent was good?

9. A farm is divided as follows: 4 acres in home lot and buildings, 20 acres in pasture, 40 acres in grass, 10 acres in alfalfa, 32 acres in corn, 16 acres in oats, 8 acres in wheat, 24 acres in potatoes, and 6 acres in miscellaneous crops. What per cent of the farm is given to each?

10. It is 90 miles from New York to Philadelphia, 95 miles from Philadelphia to Baltimore, and 40 miles from Baltimore to Washington. In going from New York to Washington, what per cent of the trip is completed when Philadelphia is reached? Baltimore? Washington?

11. What is the per cent of gain in buying apples at 50 cents a bushel and selling at 80 cents a bushel?

HINT. What is the gain? The gain is what part of the cost?

The per cent of gain or loss is always reckoned on the cost.

12. A man bought a cow for \$80 and sold it for \$60. What was his per cent of loss?

13. A market man bought cranberries at \$2.50 per bushel and sold them at 80 cents a peck. What per cent did he gain?

14. What is the per cent of profit when 5 plows are bought for \$75 and sold at \$18 apiece?

15. A drover bought 8 cows at \$62.50 apiece, and sold them for \$620. What was his gain per cent?

16. A box of mucilage (3 dozen bottles) is bought for \$1.26 and retailed at 5 cents a bottle. What is the gain per cent?

FINDING THE WHOLE, OR THE BASE

Solve:

1. 12 is 25% of ——.
2. 10 is 20% of ——.
3. 18 is 50% of ——.
4. 8 is $16\frac{2}{3}\%$ of ——.
5. 15 is 75% of ——.
6. 16 is 40% of ——.
7. 20 is $33\frac{1}{3}\%$ of ——.
8. 48 is $66\frac{2}{3}\%$ of ——.
9. 9 is $12\frac{1}{2}\%$ of ——.
10. 30 is $83\frac{1}{3}\%$ of ——.
11. 15 is $37\frac{1}{2}\%$ of ——.
12. 36 is 60% of ——.
13. 25 is $62\frac{1}{2}\%$ of ——.
14. 28 is $87\frac{1}{2}\%$ of ——.

ORAL PROBLEMS

1. Arthur sold 12 of his doves, or 25% of his flock. How many had he at first?

2. \$30 is $16\frac{2}{3}\%$ of the money Albert has in the bank. How much has he in the bank?

3. Two weeks, or $12\frac{1}{2}\%$, of the school term have passed. How many weeks in the term?

4. Mr. Wells sold 36 tons of alfalfa, or 75% of his crop. How many tons did he raise?

5. A farmer sold 20% of his stock. He then had 24 head. How many had he at first?

6. Mr. Watson has 60 sheep, or 80% of his flock, in the pasture, and the rest in the barn. How many in the barn?

7. A suit cost $27. This was 90% of the cost of a coat. Find the cost of the coat.

WRITTEN PROBLEMS

8. A farmer sold his horse at a gain of $30, or 25%. Find the cost.

(1) $30 is $\frac{1}{4}$ of the cost,
$\frac{4}{4}$, or the whole, is 4 times $30, or $120.

$$\begin{array}{l} \qquad\qquad\qquad\quad \$120 \\ (2)\ .25\overline{)\$30} = 25\overline{)\$3000}. \end{array}$$

9. When Mr. Blackburn sold his cow for 15% more than he paid, he gained $12. How much did the cow cost?

10. A mowing machine was sold at a loss of $14, or 40%. Find the cost.

11. If Mr. Peters sells his house for $4500, he will gain 20% of the cost. Find the cost.

HINT. $4500 = what per cent?

12. A clerk had his weekly wages increased $3, or $16\frac{2}{3}$%. What were his wages before the increase? After the increase?

13. A merchant sold 123 barrels of flour, or 82% of his stock, in one day. How many barrels had he at first?

14. What is the cost of a fountain pen if a profit of 15% is made by selling it for 69 cents?

15. Find the cost of a piano sold at a profit of $108, or 45%.

16. What wages does a man receive weekly if he spends 80%, or $19.20, for living expenses?

17. If 65% of a barrel of sugar is 195 pounds, what is the weight of a full barrel?

18. A ranchman lost 3000 sheep in a blizzard. If this was 24% of his herd, how many had he at first?

19. A plow sold for $15 gives a profit of 25%. Find the cost.

20. A harrow sold for $15 gives a loss of 25%. Find the cost.

21. A farmer sold two cows for $72 each. On one he gained 20%, and on the other he lost 20%. Did he gain or lose by the sale? How much?

ORAL EXERCISE

Find the missing term:

	Cost	Selling Price	Gain	Loss	Gain %	Loss %
1.	$24		?		12½	
2.	$40	?				20
3.	$30		$5		?	
4.		$36		$4		?
5.	?		$7		20	
6.	$27	$36			?	
7.		$32	$8		?	
8.	?	60			25	

WRITTEN EXERCISE

Find the missing term:

	Cost	Selling Price	Gain	Loss	Gain %	Loss %
1.	$120		?		15	
2.	$350			?		8
3.	$270	$360			?	
4.	$147	$140				?
5.		$630	$70		?	
6.		$225		$25		?
7.	$360	?			$16\frac{2}{3}$	
8.	$240	?				20
9.	$12.50	?			32	
10.	$18.25	$21.90			?	
11.		$21	$4.20		?	
12.	$2.80	?			$37\frac{1}{2}$	
13.	$6.50			?		18
14.	$8.40	?				15
15.		$42		$4.50		?
16.	$812	?				7

MISCELLANEOUS PROBLEMS

1. Mr. Barker paid $4875 for a house which he sold a month later for 8% more than he paid. Find the selling price.

2. An alloy is composed of 18 parts silver and 7 parts copper. What per cent of the alloy is silver? What per cent copper? How many pounds of each metal in an alloy weighing 1200 pounds?

3. What is the loss when an automobile that cost $1800 is sold for 58% of the cost?

4. The population of a city in 1909 was 12,800; in 1910, 5 % more; in 1911, 5 % more than in 1910. Find the population in 1911.

5. Mr. Sayles bought a cow for $84 and sold it at a profit of $12. What was his gain per cent?

6. A merchant bought 45 rugs for $113.75, and sold them at a gain of 20 %. How much did he receive for each rug?

7. In a city of 80,000 people 20 % are of school age, but only 80 % of the children of school age attend school. How many pupils in school?

8. A house lot costing $1750 was sold for 92 % of the cost. Find the selling price.

9. Gunpowder is 75 % saltpetre, 10 % sulphur, and 15 % charcoal. How many pounds of each material must be used to make a ton of gunpowder?

10. A watch costing $42 was sold at a loss of $12\frac{1}{2}$ %. What was the selling price?

11. A bricklayer formerly laid 1000 brick a day. By a scientific handling of materials, he can now lay 2700 a day. What is the per cent of increase?

12. Mr. Adams bought a house for $3750 and sold it at a gain of $16\frac{2}{3}$ %. Find the selling price.

13. What is the gain per cent when wood is bought for $6.75 per cord and sold for $8 per cord?

14. A salesman receives a salary of $1200 and 5 % on all sales. What is his income in a year in which he sells $18,000 worth of goods?

15. A platform scale sold for $13.94 allows a profit of $3.69. What per cent?

16. How much must a dealer in farm supplies get for a hay rake costing $16.75 to make a profit of 30 %?

17. Henry buys newspapers at the rate of 2 for a cent and sells them at a cent apiece. Find his gain per cent.

18. A cotton mill has in its storehouse 4284 bales of cotton. If it uses 16⅔ % of its stock in a week, how long will the supply last? How many bales are used each week?

19. What is the loss in selling a horse that cost $165 for 7 % less than cost?

20. Mr. Appleton paid $27.50 for a disc cultivator, and at the end of the season sold it for $16.50. What per cent of the cost did he receive?

21. A man built a house for $3750 and sold it for $675 above cost. What per cent did he gain?

22. Twelve per cent of an alloy is tin, and the rest copper. How many pounds of copper in 245 pounds of the alloy?

23. A galvanized steel grain tank that can be made for $24 is sold for $32.60. What per cent does the dealer make?

24. A picture dealer paid $18 for a picture and $6 for the frame. He sold both at a gain of 45 %. Find the selling price.

25. A farmer bought 60 sheep at $4.50 apiece and sold them for $360. What was the gain per cent?

26. If green coffee loses 8⅓ % of its weight in roasting, how many pounds of roasted coffee will be produced from 375 pounds of green coffee?

27. What is the gain per cent in buying wheat at 90 cents a bushel and selling at $1.05 a bushel?

28. A barrel of flour (196 pounds) is put up in bags of 24½ pounds. A bag of flour is what per cent of a barrel?

29. "Kind hearts are more than coronets,
And simple faith, than Norman blood."

What per cent of the words in this couplet are words of 3 letters? Of 4 letters? Of 5 letters? Of 6 letters? Of 8 letters?

REAL ESTATE PROBLEMS

Mr. Ballard bought a lot of land for $1200 and built a 4-tenement house upon it for $4800.

1. The first year after it was built he rented each tenement for $25 a month, and paid $75 for taxes and insurance. What was the gross income? The net income? The net income was what per cent of the cost?

2. The second year he rented each tenement for $24 a month, paid $75 for taxes and insurance, and $27 for repairs. What was his net income? This was what per cent of the cost?

3. The third year he rented one tenement for the year at $23 a month, one for 8 months at $20 a month, one for the year at $20 a month, and one for 10 months at $23 a month. He paid $75 for taxes and insurance, and $69 for repairs. The net income was what per cent of the cost?

4. The fourth year he received an average rental of $75 a month from the property, and paid $75 for taxes and insurance, and $225 for repairs. What per cent income?

5. The fifth year he received an average monthly rental of $17.50 from each tenement, and paid $75 for taxes and insurance, and $285 for repairs. What per cent income?

6. At the end of the fifth year he sold the property at auction for $5800, and paid the auctioneer $100 for his services. What per cent of the cost did he receive?

7. He then bought a farm of $60\frac{5}{8}$ A. at $72 an acre, and spent a sum equal to 20 % of the cost in repairs. Find the total amount expended.

8. The following year he sold the farm at a profit of $\frac{2}{9}$ of the total cost. How much did he receive for it?

BASEBALL PROBLEMS

The regulation baseball "diamond" is a 90-foot square.

1. Draw one on a scale of 1 inch to 30 feet. Mark the home plate *H*, and the bases 1, 2, 3.

2. How many square feet does the infield cover? What part of an acre?

3. How far does a player run in making a "home run"? What part of a mile?

4. The record time for a "home run" is 14 seconds. How many feet in a second?

5. A "home run" is how much longer than a hundred-yard dash?

6. The record time for a hundred-yard dash is $9\frac{3}{5}$ seconds. How many feet in a second?

7. The pitcher's "plate" is a rectangle 6 inches by 2 feet, the long side toward the home plate. What is its area in square feet? In square inches?

8. The distance from the front of the pitcher's "plate" to the rear of the home plate is 60.5 feet. The distance from the rear of the pitcher's "plate" to the center of second base is 66 feet 4 inches (nearly). How long a throw from "home" to "second"?

9. A ball is batted to "second," caught and thrown to "first," then thrown "home." How far has the ball gone?

10. In a certain number of games a left fielder had 36 chances and accepted 32. What was his fielding average?

11. Out of 132 times at bat, a player made 33 hits. What was his batting average?

Final Standing National League — 1910

Clubs	Chicago	New York	Pittsburg	Philadelphia	Cincinnati	Brooklyn	St. Louis	Boston	Games Won
Chicago	..	14	12	14	16	16	15	17	
New York	8	..	12	15	14	14	12	16	
Pittsburg	10	10	..	11	12	12	17	14	
Philadelphia	8	7	11	..	12	13	10	17	
Cincinnati	6	8	10	10	..	15	12	14	
Brooklyn	6	8	10	9	7	..	12	12	
St. Louis	7	10	4	12	10	10	..	10	
Boston	5	6	8	4	8	10	12	..	
Games Lost									

12. Make a table showing the number of games lost, the number won, the total number played, and the per cent won by each club. Express the per cent to the nearest tenth. Thus, for Chicago, $\frac{104}{154} = .675 = 67.5\%$.

National League Statistics — 1910

Clubs	Times at Bat	Runs	Hits	2-Base Hits	3-Base Hits	Home Runs	Total Bases
Chicago	4977	711	1333	219	84	34	
New York	5061	715	1391	204	83	31	
Pittsburg	5125	655	1364	214	83	33	
Philadelphia	5171	674	1319	223	71	22	
Cincinnati	5121	620	1326	150	79	23	
Brooklyn	5125	497	1174	166	73	25	
St. Louis	4912	637	1217	167	70	15	
Boston	5123	495	1260	173	49	31	
Totals							

13. Find the total number of times at bat; runs made; base hits; 2-base hits; 3-base hits; home runs.

14. Of the 1333 hits made by the Chicago team, 209 were 2-base hits, 84 were 3-base hits, and 34 were home runs. Find the total number of bases won by hits by the Chicago team; by each of the other teams.

15. Baseball scorers find the batting per cent of a team by dividing the number of hits by the number of times at bat. Thus, the batting per cent of the Chicago team is found by dividing 1333 by 4977. Find the batting per cent of each team.

DISCOUNT

Oral

Mr. Wilson owes the doctor $50. At the time of payment the doctor deducts 10% of the bill. How much is deducted?

For payment at the time of purchase a dry-goods dealer makes a deduction of 3 % from the amount of the bill. What deduction does he make on a bill of goods amounting to $200?

A carriage maker asks $60 for farm wagons. He makes a deduction of 20 % to other dealers. This is how much less than the regular price?

Any deduction from a regular or fixed price or amount is a *discount.*

Discounts are generally a per cent of the established price or amount.

The per cent of discount is the *rate of discount.*

Manufacturers and wholesalers issue price lists of their goods from which deductions are allowed. Such deductions are *commercial* or *trade discounts.*

Discounts are generally spoken of as so many "per cent off." Thus, a manufacturer speaks of the prices of plows as "$12, 25 % off," meaning that he will deduct 25 % from the regular price, $12, making the price to the retailer $9.

The amount of the bill after the deduction is made is the *net* price.

Find the discount:

1. $50, 2 % off
2. $70, 10 % off
3. $30, 40 % off
4. $40, 5 % off
5. $48, $16\frac{2}{3}$ % off
6. $35, 20 % off
7. $36, $33\frac{1}{3}$ % off
8. $56, $12\frac{1}{2}$ % off
9. $32, 25 % off
10. $40, $37\frac{1}{2}$ % off
11. $60, 20 % off
12. $50, 8 % off

13. The regular price of a chair is $12. I can buy it 25 % less than the regular price. What is the discount? The net price?

14. At 20 % off, what is the discount on a bill of goods amounting to $30?

15. Find the net price of a $48 mowing machine, discount 16⅔ %.

16. What is the net price of 5 books at $2 each, 20 % off?

17. Lace marked $0.40 a yard is sold at a discount of 12½ %. What is the discount?

WRITTEN PROBLEMS

1. Two boys bought a tent. The list price was $24, with a discount of 30 %. How much did each boy pay?

2. The list price of an automobile was $2250. If a discount of 16 % was allowed, what was the net price?

3. A plumber's bill was $176.50. For cash payment there was a discount of 2 %. What was the net amount of the bill?

4. The list price of a piano was $450. It was sold at a discount of 40 %. What was the selling price?

5. On a stove listed at $42 a discount of 22 % was allowed. What was the net price?

6. Find the net price of 60 pairs of blankets at $2.50 a pair, with a discount of 15 %.

7. A shopworn set of histories was sold at 40 % off. The original price was $25. What was the sale price?

8. A clothier advertises suits marked $12.25 to be sold at a discount of 20%. How much does he expect to get for a suit?

9. A carload of hay containing 12¾ tons was bought at $16 a ton, with a discount of 5% for immediate payment. What was the net price?

10. A coöperative grocery allows its shareholders a discount of 5% from the regular prices. What will a shareholder pay for a barrel of flour at $6.40 and 3 tons of coal at $7.20 a ton?

COMMISSION *Oral*

Mr. Harris sends 40 barrels of apples to Mr. White to sell for him. Mr. White sells them at $3 a barrel. What is the selling price? For making the sale Mr. White receives 5% of the selling price. How much does he receive? How much does Mr. Harris get?

The person for whom business is transacted is the *principal.*

The person transacting business for another is the *agent.*

The amount for which the goods are sold is the *gross proceeds.*

The amount paid to an agent for his services is the *commission* or *brokerage.*

The amount left after the commission and other charges are paid is the *net proceeds.*

1. At 5% commission, how much is paid an agent for selling a horse for $200? How much does the owner get?

2. Fred sells a box of soap among the neighbors for $40 at 20 % commission. Find his commission. How much does the maker get?

3. How much does Mr. Lake receive for selling a $3000 automobile at 12 % commission? Find the net proceeds.

4. Mr. Harvey paid a real estate agent $100 for selling a house for $2000. What per cent commission did he pay? What were the net proceeds?

5. What are the net proceeds when a commission merchant sells goods for $600 at 4 % commission?

6. An auctioneer sold a farm for $3500 and received $350 for his services. Find the per cent of his commission.

7. What were the gross proceeds of a sale of wheat if the net proceeds were $2400 and the commission $30?

WRITTEN PROBLEMS

1. At 1¾ % commission, find the amount paid an agent for selling 1675 bushels of wheat at 92 cents a bushel.

2. A real estate agent collects rents to the amount of $1248. If his commission is 2½ %, what per cent of the collections is returned to the owner of the property? How much money?

3. A bill collector collected the following amounts in one week: $37.20, $42.50, $20.68, $15.86, $12.35, $122.76, $86.42, $95.27, $39.48, and $112.50. Find his commission at 7½ %.

4. At a commission of 5 % of the sum collected, how much does an agent receive for collecting 75 % of a bill of $168?

5. A commission dealer sells 360 barrels of apples at \$1¾ a barrel, receiving a commission of 3%. Find the commission; the net proceeds.

6. Mr. Hanley sent to a commission firm 450 bushels of corn and 620 bushels of potatoes. The corn was sold at 45 cents a bushel and the potatoes at 70 cents a bushel. The firm charged a commission of 3%, and \$8.72 for cartage. How much money was sent to Mr. Hanley?

7. What is the per cent commission when an agent receives \$150 for selling 240 cases of goods at \$12.50 a case?

8. Mr. Carter paid \$20 to Mr. Driver for selling 2 cows at \$75 each, and a horse for \$100. Find the per cent of the gross proceeds paid to Mr. Driver.

9. After selling a lot of land for \$3600, the agent kept \$288 for his services. This was what per cent of the sum received for the land?

10. An architect receives a commission of 3½% for preparing the plans, and a commission of 2½% for superintending the construction, of a building. What is his commission if the building costs \$87,500?

11. The net proceeds of a sale of farm products was \$672.32; the commission \$91.68. Find the rate of the commission.

A GROUP OF SCHOOL PROBLEMS

1. In the Washington school there are 358 boys and 89 more girls than boys. How many pupils in the school?

2. The attendance was 784 on Monday, 776 on Tuesday, 769 on Wednesday, 775 on Thursday, and 766 on Friday. What was the average daily attendance?

3. There are 18 rooms in the building. Four have 54 seats each, six 48 seats each, and the rest 45 seats each. How many seats in all?

4. One hundred eighty-four tons of coal were required to heat the building. How much did it cost at $5.68 a ton?

5. One of the janitors received $2.25 a day and the other $1.95 a day. Each worked 308 days in the year. How much was paid them?

6. In the Lincoln school there were 648 pupils in September. During the year 38 new pupils entered from out of town, 16 from other schools in town, 25 were transferred to other schools, 9 left on account of illness, and 36 to go to work. How many pupils at the close of the year?

7. John's marks for the year were 84 in arithmetic, 76 in grammar, 92 in spelling, 87 in geography, and 88 in history. What was his average?

8. Henry was absent 6 sessions in the fall, 5 in the winter, and 4½ days in the spring. What part of the year of 36 weeks was he present? (Two sessions each day.)

9. There were 132 pupils in the fifth grade, and 119 in the sixth. How many in each grade after 17 were promoted from the fifth grade to the sixth, and 7 new pupils had entered the fifth grade and 4 had left the sixth grade?

10. Eight of the pupils lived outside the city, and each paid a tuition fee of $12 for each of the three terms. How much did they all pay in the year?

11. Maud's mark for attendance was 95 %. How many of the 360 sessions was she present?

12. Esther was present 360 sessions out of the whole number of 360 sessions. What was her per cent of attendance?

13. Ellen was absent 27 sessions. What was her per cent of attendance?

14. Each of the 98 pupils in the seventh grade purchased a geography for $0.95, an arithmetic for $0.65, and a grammar for $0.56.

(*a*) How much did each pupil pay for his books?

(*b*) How much did all the geographies cost?

(*c*) How much did all the arithmetics cost?

(*d*) How much did all the grammars cost?

(*e*) What was the total amount paid by all the pupils? Find this in two ways.

15. The average expense per pupil for books and supplies was $1.87. What amount was expended for the pupils in the Washington school?

16. The pupils in a school are divided among the grades as follows: 180 in the fourth grade, 150 in the fifth, 120 in the sixth, 100 in the seventh, and 50 in the eighth. What part of the pupils in the school in each grade?

17. In another school there are 720 pupils. Ten per cent are in the eighth grade, 15 % in the seventh grade, 20 % in the sixth grade, 25 % in the fifth grade, and the rest in the fourth grade. How many in each grade?

18. In the Lee school there are 90 pupils in the eighth grade, 30 more in the seventh grade, 20 more in the sixth grade than in the seventh, 40 more in the fifth grade than in the sixth, and 40 more in the fourth grade than in the fifth. How many in each grade? In the school? What per cent of the pupils in the school in each grade?

19. Of the 64 pupils in the highest grade of a grammar school, $87\frac{1}{2}\%$ received certificates of admission to the high school, and 75 % of those receiving certificates entered the high school. How many entered the high school?

20. The length of the high school session is 5 hours. Thirty minutes are given to recesses, and the rest of the time is divided into 6 equal periods. How many minutes to a period? What per cent of an hour?

DIFFERENCE IN TIME BETWEEN DATES

Find the difference in time between March 22, 1910, and Nov. 12, 1912.

	Years	Months	Days
(1)	1912	11	12
	1910	3	22
	1	7	20

In this method a month is regarded as 30 days.

(2) From March 22, '10, to March 22, '11 = 1 year
From March 22, '11, to Oct. 22, '11 = 7 months
From Oct. 22, '11, to Nov. 12, '11 = 21 days

This method gives the exact number of days less than one month.

Both the above methods are in common use. Use the one generally used by business men in your locality.

TABLE OF DAYS GENERALLY OBSERVED AS HOLIDAYS

New Year's Day,	January 1.
Washington's Birthday,	February 22.
Peace Day,	May 18.
Flag Day,	June 14.
Independence Day,	July 4.
Labor Day,	First Monday in September.
Columbus Day,	October 12.
Thanksgiving Day,	Last Thursday in November.
Christmas Day,	December 25.

Find the time in months and days:

1. From New Year's Day to Christmas Day.
2. From Washington's Birthday to Peace Day.
3. From Washington's Birthday to Independence Day.
4. From Independence Day to the following Flag Day.
5. From Peace Day to Christmas Day.
6. From Christmas Day to the following Peace Day.
7. From to-day to next New Year's Day.
8. From to-day to each of the other days in the above list.

Find the difference in time between the following dates:

1. April 6, 1904, and Nov. 1, 1909.
2. Sept. 27, 1906, and March 3, 1909.
3. Nov. 28, 1905, and Aug. 15, 1909.
4. Jan. 13, 1906, and Aug. 7, 1911.
5. May 12, 1902, and Jan. 3, 1909.
6. Oct. 17, 1911, and Nov. 7, 1912.
7. Feb. 5, 1905, and March 3, 1908.
8. Dec. 29, 1910, and May 16, 1913.
9. March 21, 1906, and Aug. 17, 1911.
10. July 20, 1904, and Jan. 26, 1907.
11. Aug. 24, 1900, and March 12, 1912.
12. June 27, 1910, and April 5, 1913.

SIMPLE INTEREST

Mr. Campton owns a store worth $4000. He lets Mr. Barker have the use of it for one year for 6% of its value. How much does Mr. Barker pay?

If Mr. Campton should let Mr. Barker have $4000 in money for a year at 6%, how much would Mr. Barker pay for its use?

Money paid for the use of money is interest.

The sum of money for whose use interest is paid is the principal.

The per cent of the principal paid each year for interest is the rate.

The *legal* rate is the rate established by law in the state where the money is loaned. The legal rate in most states is 6%. What is the legal rate in the state in which you live? When no rate is mentioned, the legal rate is understood.

Mr. Stevens borrowed $3000 for one year at 5%. At the end of the year he paid the $3000 and the interest. What amount did he pay?

The sum of the principal and the interest is the amount.

SIGHT EXERCISES

Find the interest for one year on:

1. $300 at 6%
2. $800 at 5%
3. $900 at 4%
4. $700 at 3%
5. $800 at 7%
6. $500 at 8%
7. $250 at 6%
8. $150 at 8%
9. $350 at 4%
10. $240 at 5%
11. $450 at 3%
12. $900 at 7%

Find the interest on:

13. $150 at 4% for 1 year; ½ of a year; 1½ years.

14. $250 at 8% for 1 year; ¼ of a year; ¾ of a year; 1¼ years.

15. $450 at 6 % for 1 year; $\frac{1}{3}$ of a year; $\frac{2}{3}$ of a year; $1\frac{2}{3}$ years.

16. $600 at 5 % for 1 year; $\frac{1}{6}$ of a year; $\frac{5}{6}$ of a year; $1\frac{5}{6}$ years.

17. What part of a year is 6 months? 3 months? 9 months? 4 months? 8 months? 2 months? 10 months? 1 month? 5 months? 7 months? 11 months?

18. What is the interest on $400 at 6 % for 6 months? 3 months? 9 months? 4 months? 8 months? 2 months? 10 months? 1 month? 5 months? 7 months? 11 months?

Find the interest and the amount of:

19. $800 for 6 months at 5 %

20. $600 for 3 months at 8 %

21. $900 for 1 month at 4 %

22. $600 for 8 months at 3 %

WRITTEN PROBLEMS

1. What is the interest and the amount of $576 for 2 years 7 months at 5 %?

$576, principal
.05
$28.80, interest for 1 year
$2\frac{7}{12}$ (2 years 7 months = $2\frac{7}{12}$ years)
$16.80
57.60
$74.40, interest for 2 years, 7 months
576.00, principal
$650.40, amount

Find the interest and the amount of:

2. \$490 for 1 year 9 months at 7 %

3. \$867 for 2 years 4 months at 8 %

4. \$1265 for 3 years 5 months at 6 %

5. \$1468 for 4 years 6 months at 5 %

THE ONE-DOLLAR METHOD

In computing simple interest a year is generally regarded as 12 months of 30 days each.

When the time is given in years, months, and days, or in months and days, the following method may be used:

\$1 at interest at 6 % will earn

in 1 year				\$0.06.
in 2 months	($\frac{1}{6}$ of a year)	$\frac{1}{6}$ of \$0.06,	or	\$0.01.
in 1 month	($\frac{1}{2}$ of 2 months)	$\frac{1}{2}$ of \$0.01,	or	\$0.005.
in 6 days	($\frac{1}{5}$ of a month)	$\frac{1}{5}$ of \$0.005,	or	\$0.001.
in 1 day	($\frac{1}{6}$ of 6 days)	$\frac{1}{6}$ of \$0.001,	or	\$0.000$\frac{1}{6}$.

Find how much \$1 will earn at 6 % in the following periods of time:

1.	Years:	1	3	5	7	2	4	6	8
2.	Months:	2	4	6	8	10	1	3	5
3.	Months:	7	9	11	12	15	18	20	24
4.	Days:	6	12	18	24	3	9	15	21
5.	Days:	27	2	4	8	10	14	16	20
6.	Days:	22	26	28	5	7	11	13	17
7.	Days:	19	23	25	29	36	48	60	90

8. What is the interest on \$1 for 2 years 7 months 17 days at 6 %? On \$348?

At 6 % the interest on $1
for 2 years = $0.12
for 7 months = .035
for 17 days = .002$\frac{5}{6}$
for 2 years 7 months 17 days = 0.157\frac{5}{6}$
348 times 0.157\frac{5}{6}$ = ?

How much will $420 earn at 6 % in :

9. 1 yr. 6 mo.? **10.** 2 yr. 6 mo.? **11.** 1 yr. 4 mo.?

12. 1 yr. 8 mo.? **13.** 1 yr. 2 mo.? **14.** 1 yr. 10 mo.?

15. 1 yr. 2 mo. 12 da.? **16.** 1 yr. 7 mo. 11 da.?

17. 1 yr. 4 mo. 18 da.? **18.** 1 yr. 8 mo. 11 da.?

19. Find the interest on $342 for 3 months 10 days at 6 %.

20. Find the interest and the amount of $385 for 3 months 20 days at 5 %.

Interest of $1 at 6 %
for 3 mo. = $0.015
for 20 da. = .003$\frac{1}{3}$
for 3 mo. 20 da. = 0.018\frac{1}{6}$
385
6)7.058$\frac{1}{3}$, interest at 6 %
$1.176⁺, interest at 1 %
5
$5.880, interest at 5 %
385.00 , principal
$390.88 , amount

21. The interest of any principal at 5 % is what part of the interest at 6 % ?

22. What part of the interest at 6 % is the interest at 3 %? 4 %? 2 %? 1 %?

THE BANKERS' METHOD

When the time is given in days, the following method, commonly called the *bankers' method*, may be used.

Since the interest at 6% of any principal for 60 days is .01 of itself, first find the interest for 60 days, and then take such parts of this interest as may be necessary. Thus, in finding the interest at 6% of $360 for 98 days:

98 days = 60 days + 30 days + 5 days + 3 days
$360 = principal
$3.60 = interest for 60 days
1.80 = interest for 30 days ($\frac{1}{2}$ of interest for 60 days)
.30 = interest for 5 days ($\frac{1}{6}$ of interest for 30 days)
.18 = interest for 3 days ($\frac{1}{10}$ of interest for 30 days)
$5.88 = interest for 98 days

Find the interest at 6% on:

1. $960 for 90 days
2. $480 for 36 days
3. $840 for 24 days
4. $360 for 48 days
5. $720 for 84 days
6. $240 for 72 days
7. $420 for 66 days
8. $990 for 40 days
9. $540 for 87 days
10. $680 for 96 days

Find the interest:

	Principal	Rate	Time		Principal	Rate	Time
11.	$640	8%	54 days	**12.**	$860	4%	45 days
13.	$760	5%	27 days	**14.**	$790	7%	33 days
15.	$240	7%	78 days	**16.**	$750	5%	42 days
17.	$675	8%	39 days	**18.**	$972	7%	50 days
19.	$864	4%	56 days	**20.**	$246	3%	69 days

PROBLEMS

1. How much will $450 earn in 1 year 3 months at 5%?

2. At 8%, what is the interest on $570 for 1 year 7 months?

3. What is the interest on $12,000 for 15 days at 4%?

4. A man has the use of $1260 for 3 months 21 days. How much interest must he pay if money is worth 5%?

5. What will settle an account of $642 that has been running 54 days, with interest at 7%?

6. What must be paid to settle in full an account of $945 that has been drawing interest at 5% for 2 months 24 days?

7. How much interest is due Oct. 17 on a note for $1485, dated June 26, with interest at 5%?

8. Mr. Davis borrowed $387.54 on Nov. 22, 1910, agreeing to pay 7% interest. He settled the account on May 10, 1911. How much did he pay?

9. On March 12 Mr. Buxton loaned $2800 to Mr. Ray at 5%, interest to be paid every 6 months. When is the first payment due? How much must Mr. Ray pay?

10. How much interest is due Oct. 11, 1911, on a note for $459 that has been running since Jan. 28, 1911, with interest at 8%?

11. Mr. Sampson buys a horse for $156, paying $40 down and agreeing to pay the balance in 3 months with interest at 5%. How much must he pay at the end of the three months?

12. $1740 is borrowed for 20 days at 5%. How much will settle the account at the end of the 20 days?

13. How much is the interest on $750 for 75 days at 4%?

14. Find the interest on $1440 for 48 days at 7%.

WRITTEN EXERCISE

Find the interest:

	PRINCIPAL	RATE	TIME	
			Mo.	Da.
1.	$ 864	5%	3	17
3.	$ 432	4%	6	15
5.	$ 582	6%	5	18
7.	$ 648	7%	3	11
9.	$ 750	3%	4	19
11.	$ 846	4%	2	17
13.	$ 750	5%	3	27
15.	$ 354	5%	4	16
17.	$1200	8%	1	20
19.	$ 672	7%	3	8
21.	$1446	5%	2	9
23.	$2970	4%	5	22
25.	$5652	5%	3	28
27.	$3456	6%	1	19
29.	$1908	5%	2	26
31.	$3642	6%	3	5
33.	$2706	4%	1	7
35.	$6336	7%	2	9
37.	$9264	5%	3	15
39.	$8346	5%	2	27
41.	$7434	4%	5	20
43.	$4194	6%	4	10
45.	$6276	5%	3	29
47.	$2832	3%	1	14

Find the amount:

	PRINCIPAL	RATE	TIME
			Da.
2.	$ 240	4%	98
4.	$ 354	5%	48
6.	$ 236	6%	123
8.	$ 360	5%	79
10.	$ 438	5%	34
12.	$ 912	3%	92
14.	$ 930	5%	76
16.	$ 870	4%	44
18.	$1410	6%	53
20.	$ 894	5%	100
22.	$6396	8%	144
24.	$2406	5%	108
26.	$1794	7%	80
28.	$5688	6%	90
30.	$3672	5%	64
32.	$1098	5%	29
34.	$2634	4%	37
36.	$4260	7%	43
38.	$7506	4%	65
40.	$3474	5%	53
42.	$5394	7%	68
44.	$6978	5%	60
46.	$4236	5%	90
48.	$1992	4%	75

MISCELLANEOUS PROBLEMS

1. The temperature at noon on Monday was 40°. It rose 8° on Tuesday, fell 17° on Wednesday, rose 3° on Thursday, fell 16° on Friday, and rose 14° on Saturday. What was the temperature on Saturday?

2. At $9.25 per M, what is the cost of 12,400 bricks?

3. A dealer sells from a roll of ribbon 60 yd. long the following quantities: .875 yd., ⅘ yd., 3⅛ yd., 2¼ yd., 5½ yd., and 8.5 yd. How many yards has he left? What per cent of the original piece?

4. How much carpeting 27 in. wide will be required to carpet a room 15 ft. by 16½ ft., allowing 1 ft. on each strip except the first for matching?

5. A dealer puts up 192 sample packages of tea in ¾-oz. packages. How many pounds of tea did he put up?

6. Mr. Curtis bought 1⅗ A. of land for $3000. He cut it up into lots containing 32 sq. rd., each of which he sold for $800. What was his entire gain?

7. Three pipes can fill a cistern in 12 hr., 15 hr., and 20 hr., respectively. How long will it take to fill the cistern: (*a*) If the three are used together? (*b*) If the first and second are used? (*c*) If the first and third are used? (*d*) If the second and third are used?

8. At $1.68 per square yard, how much will it cost to lay a floor in a room 15½ ft. by 12 ft.?

9. How much less would it cost to cover the floor with carpeting worth $1.35 per yard?

10. A contract for a railroad requires that the rails weigh 70 lb. to the yard. How many tons of rails will lay the road, which is 3½ mi. long?

11. A room 30 ft. long, 24 ft. wide, and 12 ft. high is to be plastered at 35¢ per square yard. There are 4 doors 8 ft. by 4 ft., and 6 windows 9 ft. by 3⅔ ft. Find the cost, deducting one half of the area of openings.

12. A rug 3½ yd. by 2½ yd. leaves an uncovered space 2 ft. wide all around the room. What is the floor area of the room? How many feet of picture molding must be bought for the room?

13. How many loads of earth were removed in digging a cellar, if the earth removed when spread evenly over a lot 60 ft. by 45 ft. raised the grade 3 in.?

14. 12¼ lb. of 60-cent tea were accidentally emptied into a chest containing 7¾ lb. of 40-cent tea. What is the mixture worth per pound?

15. How many panes of glass 2 ft. by 2 ft. must a schoolroom have, whose dimensions are 40 ft. by 24 ft., in order that the lighting surface may equal ⅙ the floor space?

16. A lot of land containing 1 A. is 360 ft. long. From one end is sold a lot 150 ft. long, and from the other end a lot 90 ft. long. What part of an acre is left?

17. A furnace pipe is 2½ ft. long and 22¾ in. in circumference. Allowing 1 in. for the seam, what were the dimensions of the piece of sheet iron from which it was made?

18. At $1½ per yard, what is the cost of carpeting a room 23 ft. by 19 ft. with carpeting ¾ yd. wide?

19. At ½¢ a square foot, what will it cost to sod a lawn 75 ft. long and 40 ft. wide, if the cost of laying the sod is ⅜¢ per square foot?

20. A certain city of 100,000 inhabitants covers an area of 12½ sq. mi. How many people is this to the acre?

21. At 25¢ per cubic yard, find the cost of excavating a cellar 36 ft. long, 22 ft. wide, and 6 ft. deep, and of cementing the bottom at 15¢ per square yard.

22. $37 is $\frac{1}{4}$ of A's money and $\frac{1}{5}$ of B's money. How much more than B has A?

23. A well 20 ft. deep and 4 ft. in diameter is half full of water. How many gallons?

24. What is the area over which a cow can graze if tethered to a stake by a rope 25 ft. long?

25. A pond 120 ft. in diameter gives what area for skating when frozen over?

26. What is the interest at 5 % of $2400 for 110 da.?

27. What is the gain per cent in buying collars at 75¢ a dozen and selling them at 3 for 25¢?

28. Find the area of a triangle whose base is 4 ft. 3 in. and whose altitude is 40 in.

29. Ed paid $24.75 for his bicycle and sold it for 32 % less. How much did he receive for it?

30. Two farms were sold for $3000 each. On one 20 % was gained, and on the other 20 % was lost. Did the owner gain or lose by the transaction? How much?

31. A street $\frac{1}{2}$ mi. long and 3 rd. wide is paved with asphalt. How many square yards of paving?

32. Two trains leave Chicago at 10 o'clock in the morning. One travels east at the rate of 48 mi. an hour; the other travels west at the rate of 42 mi. an hour. How far apart are the trains at 20 min. of 3 in the afternoon?

33. What must a clothing dealer ask for goods which cost him 8\frac{3}{4}$ in order to make a profit of 20 %?

34. One pipe discharges $6\frac{3}{4}$ gal. per minute, and another discharges $5\frac{7}{8}$ gal. per minute. How long will it take to empty a tank whose capacity is 600 gal.?

35. If to a disinfecting solution is added 12 parts of water, what fraction represents the strength of the mixture?

36. On a plan drawn on the scale of $\frac{1}{2}$ in. to a foot, what is the area of a lot whose frontage is $33\frac{1}{2}$ in. and whose depth is $48\frac{3}{4}$ in.?

37. 7.7 is the product of four factors, three of which are 1, 3.08, and 2.5. What is the fourth?

38. Which covers the greater space, a tile 6 in. square, or a tile containing 6 sq. in.? How many times the smaller is the larger? What part of the larger is the smaller?

39. Mrs. Hovey bought two remnants of silk; one lacked 9 in. of being 2 yd., and the other 6 in. of being $1\frac{3}{4}$ yd. What was the total number of yards bought?

40. A heating apparatus is warranted to supply 30 cu. ft. of fresh air per minute for every person in the room. If the room is 30 ft. long, 25 ft. wide, and 12 ft. high, and contains 40 persons, how often must the air in the room be renewed?

41. How many 500-lb. bales of cotton can be harvested from a field of $168\frac{3}{4}$ A., if the average yield per acre is 240 lb.?

42. An ice boat sails 18 mi. in 1 hr. 3 min. What is the rate per minute? How long does it take to go a mile?

43. A farmer raised 900 bu. of beans. One dealer offered him 4½¢ per pound, and another dealer offered $2.40 per bushel of 60 lb. Which was the better offer? By how much?

44. From the sum of 64.007 and 3.02 take the difference between 30.04 and .0607.

45. What must be added to the sum of .0105 and .604 to make one whole one?

46. Divide the sum of .03125 and .01875 by their difference.

47. Under a street 1320 ft. long there is laid a telephone cable consisting of 48 wires. How many miles of telephone wire in the cable?

48. Washington was born Feb. 22, 1732. Franklin was born Jan. 17, 1706. How old was Franklin when Washington was born?

49. When steel rails are manufactured for $13.50 per ton, and sold for $28 per ton, a profit of $26,100 on one sale represents the sale of how many tons?

50. A block of stone 4 ft. × 3 ft. × 5 ft. is used as a watering trough. A place 2 ft. 6 in. long, 1 ft. 8 in. wide, and 1 ft. deep is hollowed out of the top. How many cubic feet of stone in the block? How many gallons of water will the top hold?

51. A quart of floor varnish will cover a space of 150 sq. ft. How much will it cost at $2.80 per gallon to varnish two floors whose dimensions are 18 ft. by 15 ft., and 15 ft. by 12 ft.?

52. A reservoir is 60 ft. 8 in. long and 48 ft. 4 in. wide. To make some repairs it is necessary to lower the water 15 in. How many cubic feet must be drawn off?

53. Wishing to measure a field, a boy used a pole which he supposed to be 10 ft. in length. He found the field to be 180 ft. by 90 ft. He afterwards found his measure to be 9 ft. long. What were the real dimensions of the field?

54. How many dress patterns of 12¾ yd. each can be cut from a piece of goods containing 85 yd.? What will be the length of the remnant?

55. By what must you multiply the sum of 40.4 and 4.04 to get 4444?

56. What is the capacity in gallons of a tank 24 in. long, 11 in. wide, and 14 in. deep?

57. On a certain day the sun shone from 9:45 to 11:30 A.M., from 1:20 to 2, and from 2:50 to 3:25 P.M. What part of the 24 hours did the sun shine?

58. At 16⅔¢ per square yard, what will it cost to paint a kitchen floor 15 ft. 9 in. long and 12 ft. wide?

59. When a dealer buys 28 bu. of peas at $1.20 per bushel and 8 bu. are found to be worthless, for what must he sell the remainder per peck to get his money back?

60. By selling a shopworn book for $1.20 a dealer sacrifices 75%. What was the cost?

61. By what must you divide the difference between .015 and .15 to get 13.5 for a quotient?

62. How many paving stones 8 in. square will be required to pave a street 20 rd. long and 60 ft. wide?

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